Source Seeking for Underactuated Vehicles

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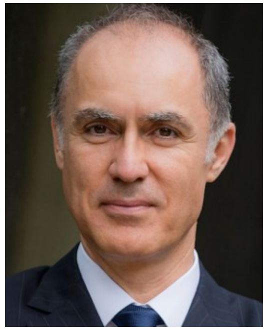
CUNY-City College

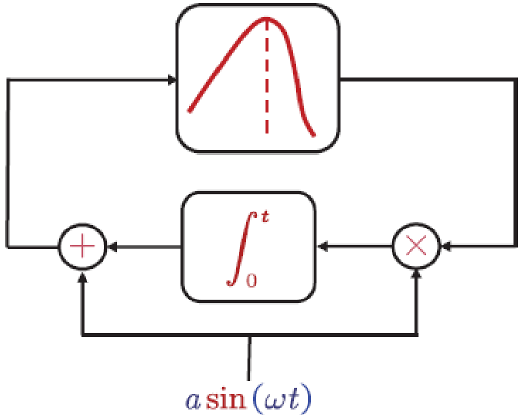
- The college has graduated 12 Nobel Prize winners, 1 Fields Medalist, 1 Turing Award winner.
- Henry Kissinger, Colin Luther Powell, Michio Kaku (加來道雄), …
- Albert Einstein gave the first of his series of United States lectures at CCNY in 1921.

Extremum Seeking Control (ESC)

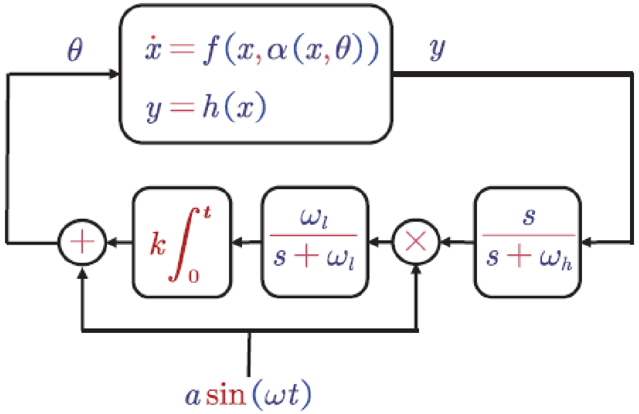
- Extremum seeking (ES)
  - Online black-box reward-seeking
  - Real-time, model-free optimization algorithm

- ES history
  - Sinusoid-based ES was popular in the 1950s
  - Stability proved in [Krstic & Wang '00]
  - Torrent of advances in theory & applications (12,000+ papers since 2000)





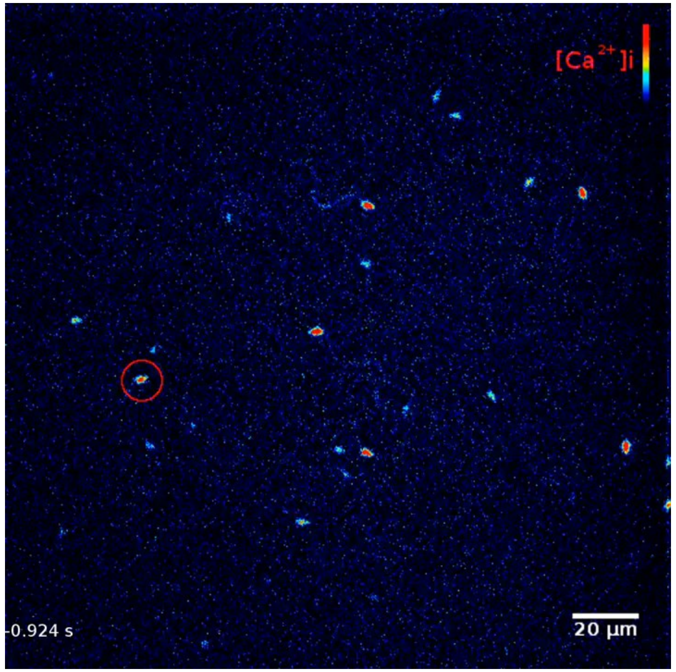
(a) ES scheme for a static map



(b) ES for a dynamical system

Krstic & Wang (2000)

Extremum Seeking in Nature



Sperm seeking an egg

Phototropism

Sea urchin sperm exploit extremum seeking control to find the egg

Mahmoud Abdelgalil[1,*], Yasser Aboelkassem[2,3], Haithem Taha[1]

[1] Mechanical and Aerospace Engineering Department, University of California at Irvine, Irvine, CA 92697
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[3] Michigan Institute for Data Science, University of Michigan, Ann Arbor MI 48109

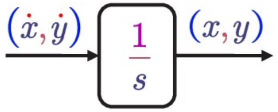
- **Motivation**: Locate the Source of a Signal (w/o GPS or INS)

- Electro-magnetic, optical, chemical, etc.
- GPS and INS signals are not available
- Locate and repair the leakage, avalanche victim search, etc.

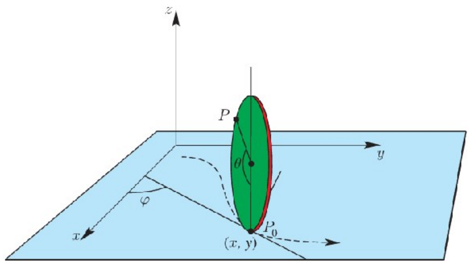
Without GPS or INS, Source Seeking algorithms are very useful!

On Vehicle Models

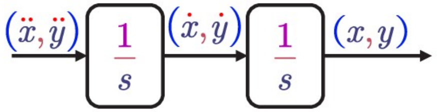
- Single/double-integrators: [Zhang'07]
- Unicycle: [Zhang'07; Ghods; Durr'13,'17]
- Fully-actuated systems: [Suttner'21,'22]
- Underactuated systems



(a) Single integrator



(c) Unicycle



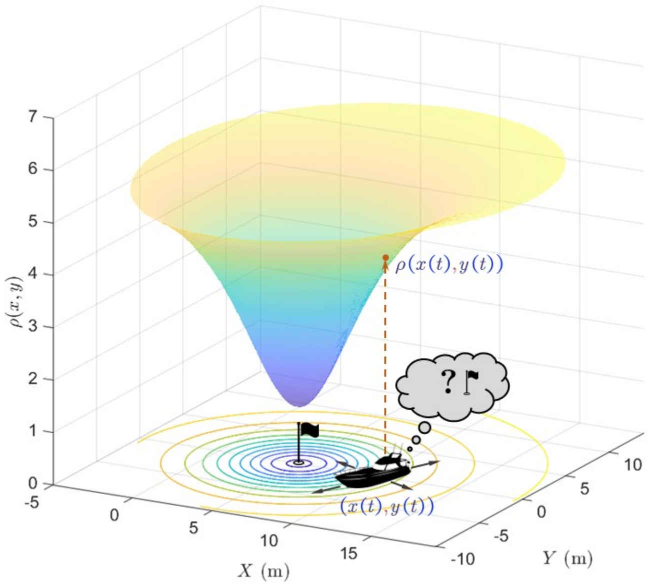
(b) Double integrator


(d) Underactuated boat

Control/Optimization Objective

- The position-dependent function ρ: $\mathbb{R}^2 \to \mathbb{R}_{\geq 0}$ has a global minimum at : $(x^\star, y^\star)$
- Both $(x^\star, y^\star)$ and $\nabla\rho$ are unknown.
- Vehicle can **only** measure the value of $\rho(x(t), y(t))$ in real time.
- Without position & velocity measurements

$$\lim_{t\to\infty} |(x(t), y(t)) - (x^\star, y^\star)| \leq \varepsilon.$$



Keyword: Underactuated Mechanical Systems

- Mechanical systems

$$\underbrace{M(q)\ddot{q} + C(q,\dot{q})}_{\text{geometric acceleration}} + \underbrace{D(\dot{q})}_{\text{damping}} + \underbrace{g(q)}_{\text{gravity}} = \underbrace{G(q)u}_{\text{external forces}}$$

  - $q \in \mathbb{R}^n$: configuration variable
  - $u \in \mathbb{R}^m$: input variable, $m < n$
  - $G(q) \in \mathbb{R}^{n \times m}$: input matrix

  - Fully-actuated, if $\text{rank}\{G(q)\} = n$
  - Underactuated, if $\text{rank}\{G(q)\} < n$

- Simple, power saving, make better robots…
- Almost all real vehicles are underactuated!

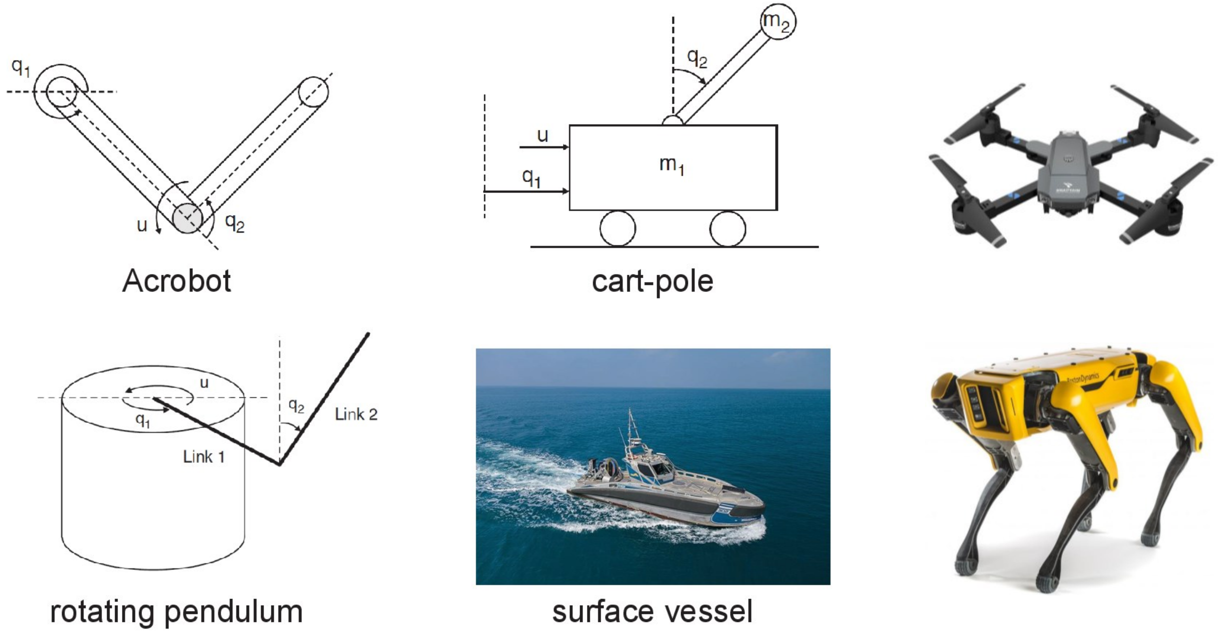


Fig: Underactuated systems.

2. Extremum Seeking Algorithms

1. Classical averaging-based (2000-2013)

PERGAMON

Automatica 36 (2000) 595–601

www.elsevier.com/locate/automatica

Brief Paper

Stability of extremum seeking feedback for general nonlinear dynamic systems☆

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Consider the system

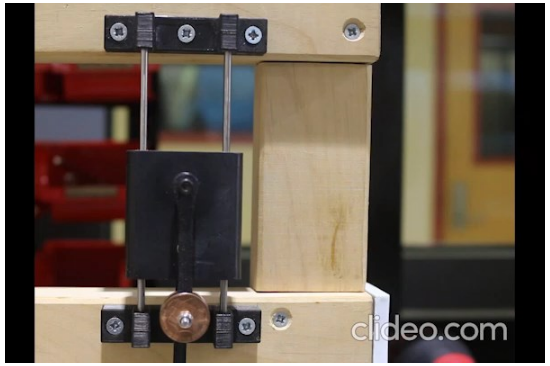
$$\dot{x} = \varepsilon f(t, x, \varepsilon) \tag{10.23}$$

where f and its partial derivatives with respect to (x, ε) up to the second order are continuous and bounded for $(t, x, \varepsilon) \in [0, \infty) \times D_0 \times [0, \varepsilon_0]$, for every compact set $D_0 \subset D$, where $D \subset R^n$ is a domain. Moreover, $f(t, x, \varepsilon)$ is T-periodic in t for some $T > 0$ and ε is positive. We associate with (10.23) an autonomous average system

$$\dot{x} = \varepsilon f_{av}(x) \tag{10.24}$$

where

$$f_{av}(x) = \frac{1}{T}\int_0^T f(\tau, x, 0)\, d\tau \tag{10.25}$$



2. Lie-bracket approximation-based (2013-)

Automatica 49 (2013) 1538–1552

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Automatica

journal homepage: www.elsevier.com/locate/automatica

Lie bracket approximation of extremum seeking systems☆

Hans-Bernd Dürr[a,1], Miloš Stanković[b], Christian Ebenbauer[a], Karl Henrik Johansson[c]

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obtain an input-affine system of the form

$$\dot{x} = b_1(x)\sqrt{\omega}u_1(\omega t) + b_2(x)\sqrt{\omega}u_2(\omega t) \tag{2}$$

with $b_1(x) = \alpha$ and $b_2(x) = f(x)$. Interestingly, if one computes the so called Lie bracket system involving $[b_1, b_2]$, i.e.

$$\dot{z} = \frac{1}{2}[b_1, b_2](z) = \frac{\alpha}{2}\nabla_z f(z), \tag{3}$$

PROCEEDINGS OF THE IEEE, JANUARY 1976

Fig. 1. Motivating the definition of ubiquitous bracket of Lie.

is the so-called *Lie bracket* (also sometimes called the Jacobi bracket, also sometimes defined with the opposite sign) of the vector fields f_1 and f_2. This calculation, which "everyone should do once in his life" is most significant. Everything else depends on it. If $[f_1, f_2]$ is not a linear combination of $f_1, f_2, \cdots, f_m$ then $[f_1, f_2]$ represents a "new" direction in which the solution can move and the original problem of finding a manifold such that $f_1, f_2, \cdots, f_m$ span the tangen space will not be solvable.

3. Symmetric-product approximation based (2021-)

SIAM J. Control Optim.
Vol. 41, No. 2, pp. 542–562

AVERAGING AND VIBRATIONAL CONTROL OF MECHANICAL SYSTEMS*

FRANCESCO BULLO†

Abstract. This paper investigates averaging theory and oscillatory control for a large class of mechanical systems. A link between averaging and controllability theory is presented by relating the key concepts of averaged potential and symmetric product. Both analysis and synthesis results are presented within a coordinate-free framework based on the theory of affine connections.

The analysis focuses on characterizing the behavior of mechanical systems forced by high amplitude high frequency inputs. The averaged system is shown to be an affine connection system subject to an appropriate forcing term. If the input codistribution is integrable, the subclass of systems with Hamiltonian equal to "kinetic plus potential energy" is closed under the operation of averaging. This result precisely characterizes when the notion of averaged potential arises and how it is related to the symmetric product of control vector fields. Finally, a notion of vibrational stabilization for mechanical systems is introduced, and sufficient conditions are provided in the form of linear matrix equality and inequality tests.

Key words. mechanical system, averaging, vibrational stabilization, nonlinear controllability

- Linear homogeneous ODE

$$y^{(n)}(x) + \sum_{i=0}^{n-1} a_i(x) y^{(i)}(x) = 0$$

Solution: $y(x) = \sum_{i=1}^{n} c_i y_i(x)$

- Linear non-homogeneous ODE

$$y^{(n)}(x) + \sum_{i=0}^{n-1} a_i(x) y^{(i)}(x) = f(x)$$

Solution: $y(x) = \sum_{i=1}^{n} c_i(x) y_i(x)$

Theorem (Variation of constants) [Bullo '02]

Consider the dynamical system

$$\dot{x} = f(t, x) + g(t, x), \quad x(0) = x_0.$$

If $z(t)$ is the solution of the (pull-back) system

$$\dot{z}(t) = \left(\left(\Phi^g_{0,t}\right)^* f\right)(t, z), \quad z(0) = x_0,$$

then the solution $x(t)$ of

$$\dot{x} = g(t, x), \quad x(0) = z(t)$$

is the solution of the original system.

Symmetric Product Approximations

- EL vehicle model

$$\dot q = J(q)v, \qquad M\dot v + C(v)v + Dv = Gu$$

- High-amplitude high-frequency input

$$u = b_0 + \frac{1}{\varepsilon}\sum_{i=1}^{m} b_i(q)w_i\left(\frac{t}{\varepsilon}\right),$$

Theorem (Variation of constants) [Bullo '02]

Consider the dynamical system

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then the solution $x(t)$ of

$$\dot x = g(t,x), \quad \boxed{x(0) = z(t)}$$

is the solution of the original system.

1. The closed-loop system **in time scale** $\tau = t/\varepsilon$:

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{bmatrix} q \\ v\end{bmatrix} = \varepsilon \underbrace{\begin{bmatrix} J(q)v \\ -M^{-1}[C(v)v + Dv - B_0]\end{bmatrix}}_{\mathrm{f}(q,v):\ \text{time-invariant}} + \underbrace{\begin{bmatrix} 0 \\ \sum_{i=1}^{m} B_i(q)w_i(\tau)\end{bmatrix}}_{\mathrm{g}(\tau,q)}$$

2. "Pull-back": $\dot z(t) = \left(\left(\Phi^{g}_{0,t}\right)^{*} f\right)(t,z) =: \mathrm{F}(t,z)$

$$\mathrm{F} = \mathrm{f} + \sum_{i=1}^{m}\begin{bmatrix} J(q)B_i(q) \\ \left(\frac{\partial \mathrm{f}_2}{\partial v}\right)B_i - \left(\frac{\partial B_i}{\partial q}\right)J(q)v\end{bmatrix}\int_0^{\tau} w_i(s_1)\mathrm{d}s_1 - \sum_{i,j=1}^{m}\begin{bmatrix} 0 \\ \langle B_i : B_j\rangle\end{bmatrix}\int_0^{\tau}\int_0^{s_1} w_i(s_1)w_j(s_2)\mathrm{d}s_2\mathrm{d}s_1$$

3. "Homogeneous": $\dot x = g(t,x)$, $x(0) = z(t)$

$$\left\{\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{bmatrix} q \\ v\end{bmatrix} = \begin{bmatrix} 0 \\ \sum_{i=1}^{m} B_i(q)w_i(\tau)\end{bmatrix}, \quad \begin{bmatrix} q(0) \\ v(0)\end{bmatrix} = \begin{bmatrix} \hat q(\tau) \\ \hat v(\tau)\end{bmatrix}\right\} \Longrightarrow [q(\tau) \equiv q(0) = \hat q(\tau)]$$

4. "Averaged pull-back": Symmetric Product System

$$\left\{\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{bmatrix} \bar q \\ \bar v\end{bmatrix} = \frac{\varepsilon}{T}\int_0^{T}\mathrm{F}(\tau,\bar q,\bar v)\mathrm{d}\tau\right\} \Longrightarrow \left\{\begin{array}{c} \dot{\bar q} = J(\bar q)\bar v, \\ M\dot{\bar v} + C(\bar v)\bar v + D\bar v = B_0 - M\sum_{i,j=1}^{m}\Lambda_{ij}\langle B_i : B_j\rangle(\bar q)\end{array}\right\}$$

$\boxed{q(\tau) \equiv \hat q(\tau) \approx \bar q(\tau)}$ i.e., (Partial) Converging Trajectories Property

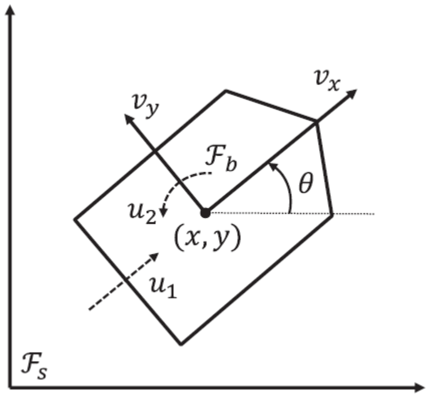
Theorem (Source seeking: surge force tuning) [Wang et al., TCST, CDC'22]

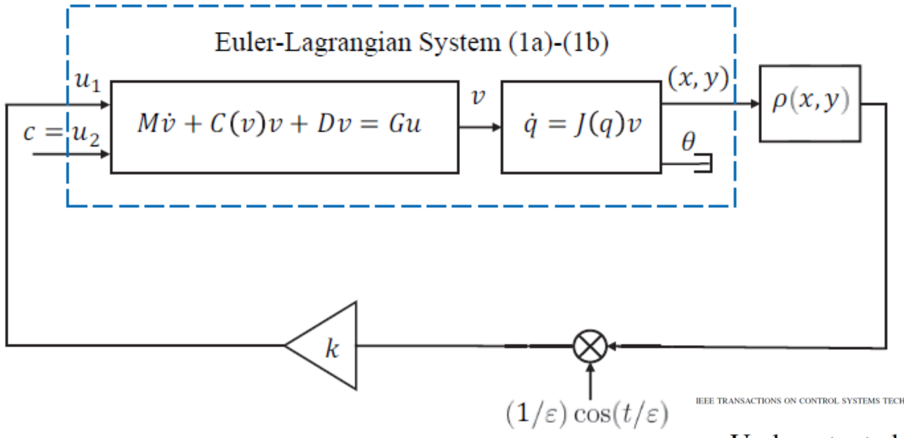
Consider the underactuated vehicle system with inputs

$$u_1 = \frac{k}{\varepsilon}\cos\left(\frac{t}{\varepsilon}\right)\rho(x,y),$$

$$u_2 = c.$$

Then, $\exists\ \hat{c} > 0$ and for any $c \in (0, \hat{c})$, $\exists\ \hat{\varepsilon} > 0$ such that for the given c and any $\varepsilon \in (0, \hat{\varepsilon})$ and $k > 0$, the closed-loop system is SPAS w.r.t. $(x - x^\star, y - y^\star, v_x, v_y)$ uniformly in $(\theta(0), \omega(0))$.





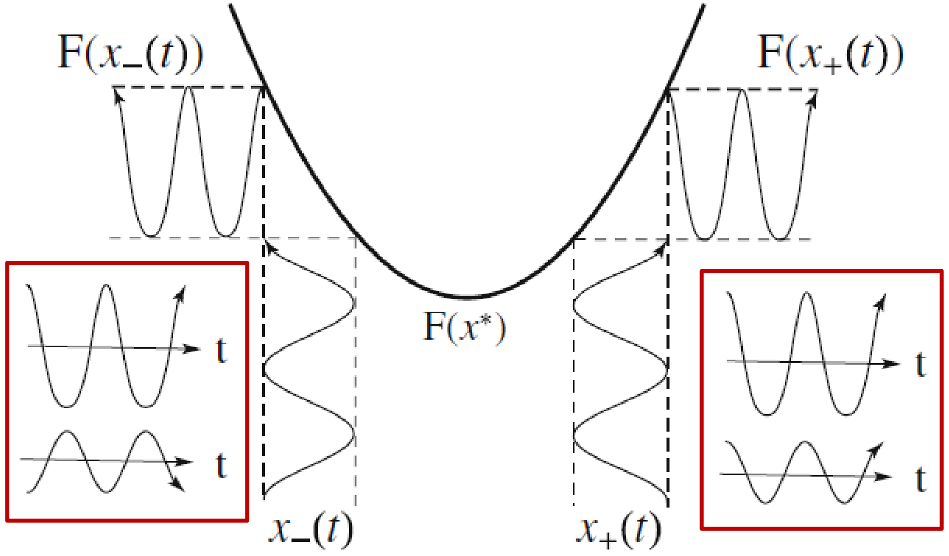

IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY

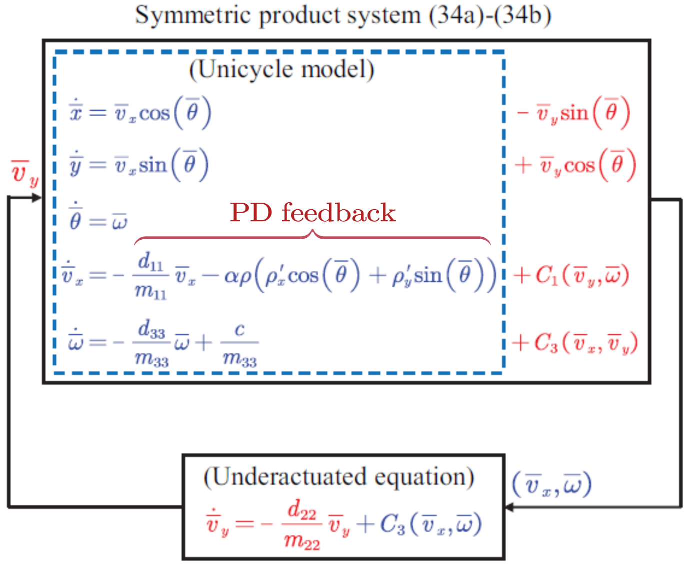
Underactuated Source Seeking by Surge Force Tuning: Theory and Boat Experiments

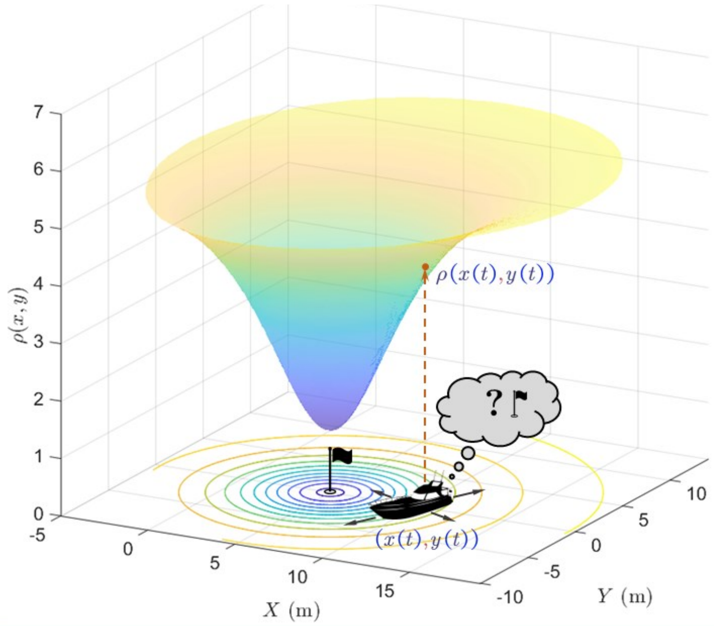
Bo Wang, Graduate Student Member, IEEE, Sergey Nersesov, Member, IEEE, Hashem Ashrafiuon, Senior Member, IEEE, Peiman Naseradinmousavi, and Miroslav Krstić, Fellow, IEEE

Key idea:

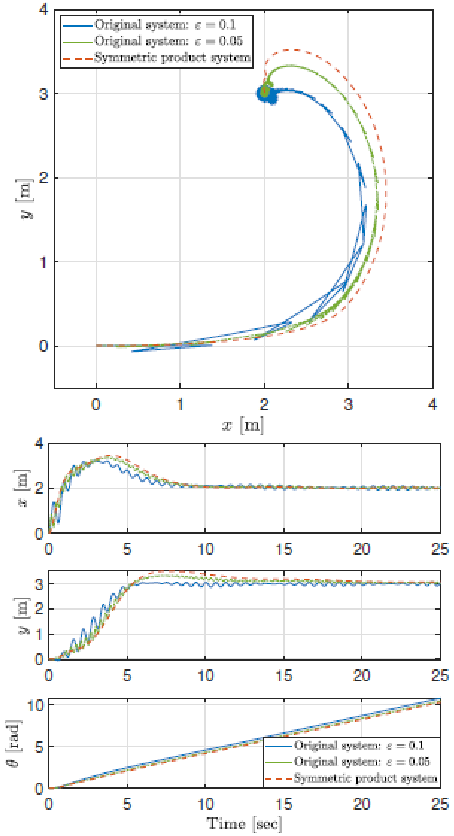
Gradient Decent?



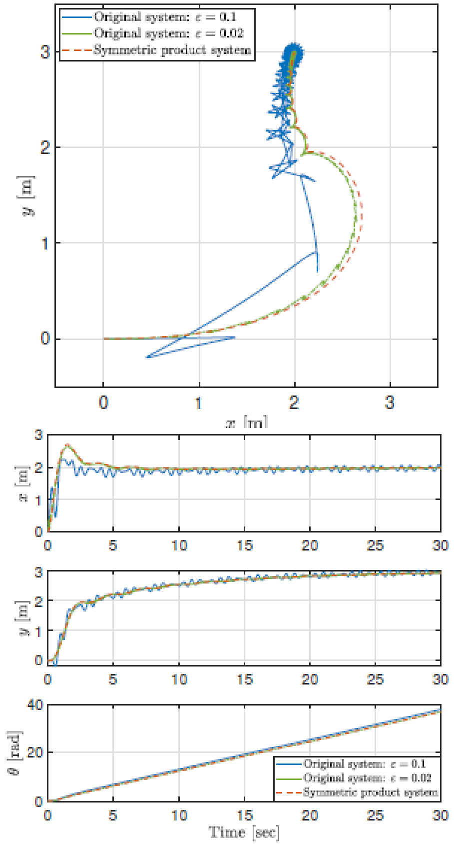




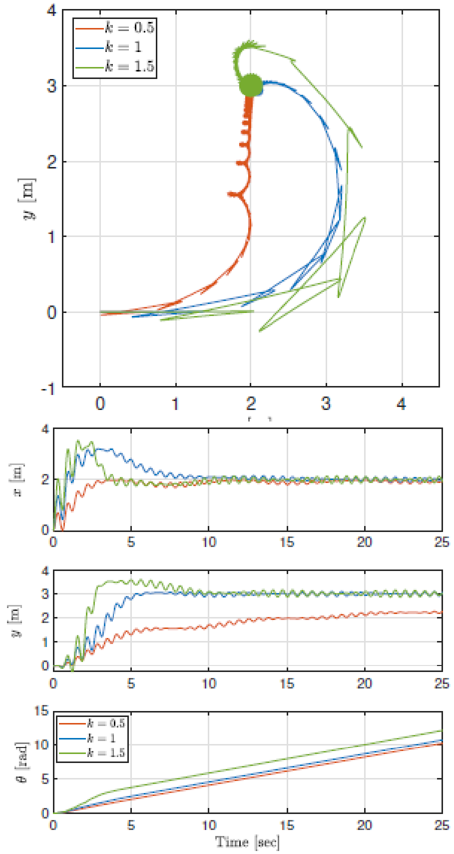
- Simulation Results: (Boat)



$(c = 1, k = 1)$

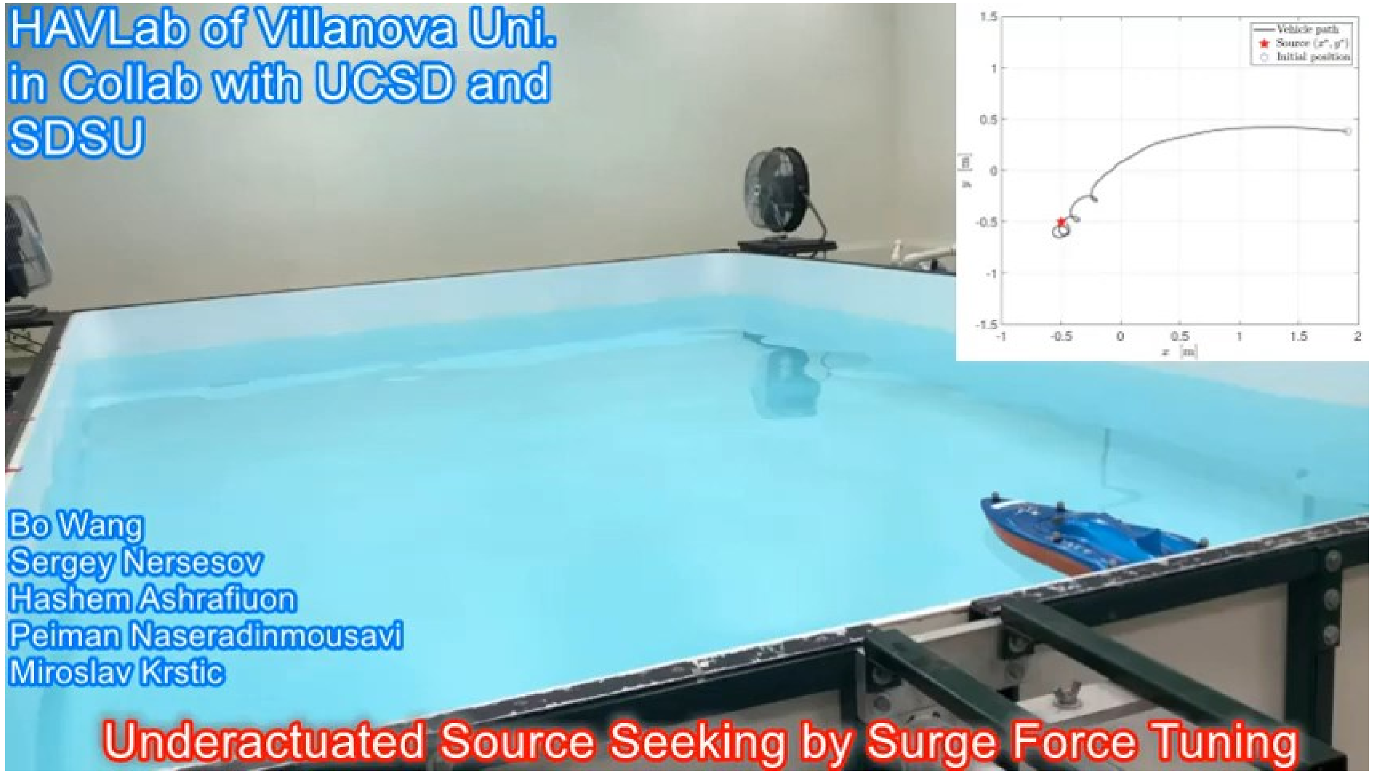


$(c = 3, k = 1)$



$(c = 1, \varepsilon = 0.1)$

- Experimental Results



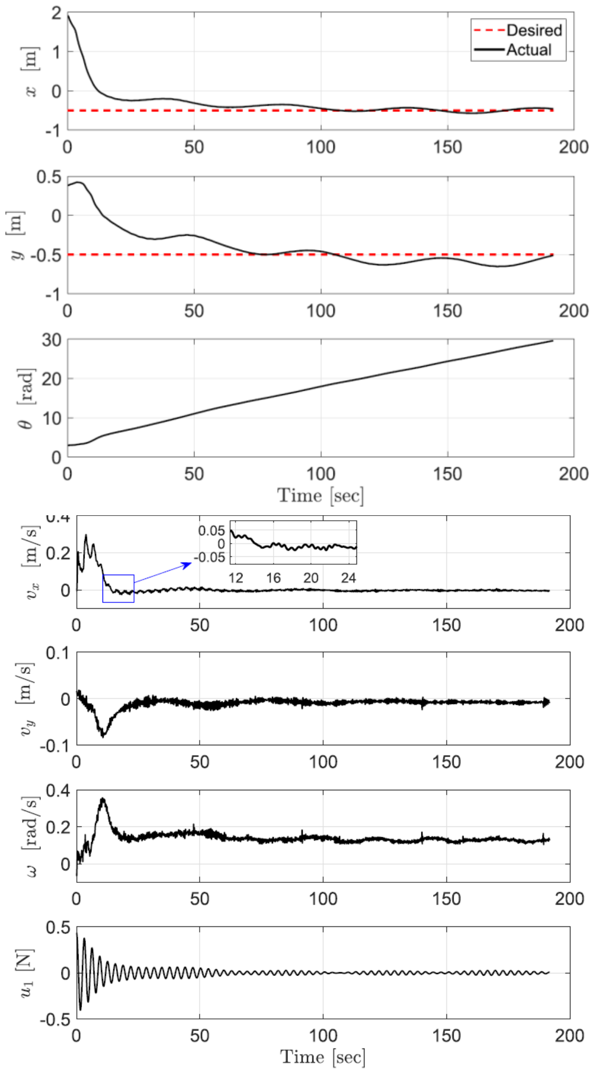
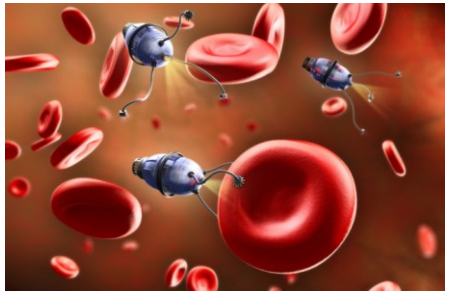
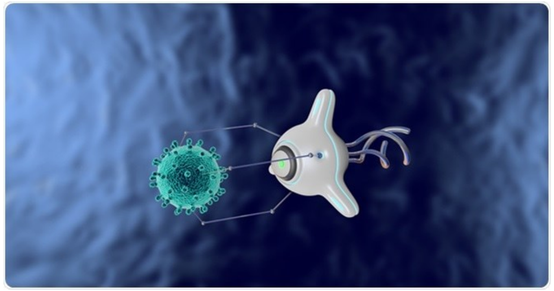


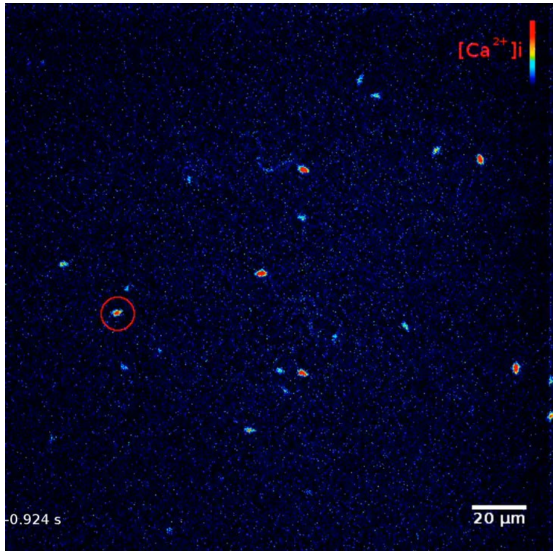
Fig: State trajectories and control input

- Medical Robotics
  - Micro/Nano Robotics with Source Seeking ability

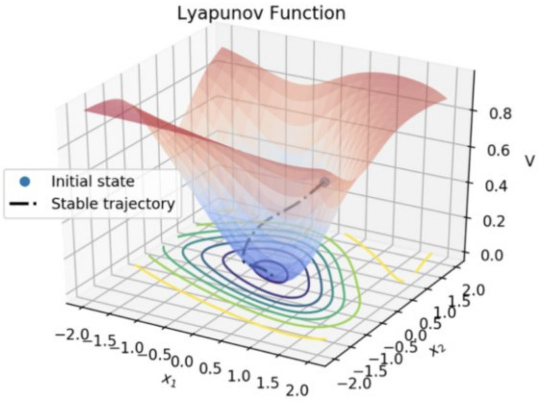
(Tumor, cancel cells seeking and treatment)



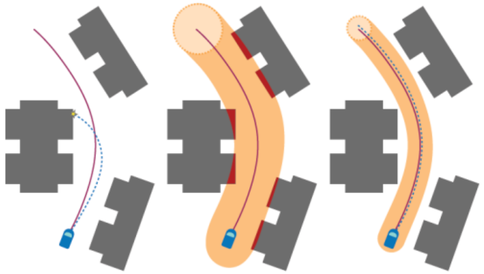




- Learning-Based Control
  - Data-driven, seeking the minimum of a “Lyapunov”



- Safety Critical Control
  - Guarantee safety in complex environment (CBF)
  - Fixed/prescribed-time control



- Looking for outstanding **Ph.D. students** interested in Control Theory and Underactuated Systems
- **Collaboration is welcome!**
  - Email: bwang1@ccny.cuny.edu / bwang.ccny@gmail.com
  - Website: https://bwang-ccny.github.io

Thank you for the attention!