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Leader–follower formation stabilization and tracking control for heterogeneous planar underactuated vehicle networks ☆

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ABSTRACT

In this paper, we solve the distributed leader–follower simultaneous formation stabilization and tracking control problem for heterogeneous planar underactuated vehicle networks without global position measurements of the followers. The vehicles in the network are modeled as generic 3-DOF planar rigid bodies with two control inputs, and are allowed to have identical or non-identical dynamics. By incorporating graph theory, passivity-based control, partial stability theory, Matrosov's theorem and the persistence of excitation concept, a smooth formation control scheme is proposed to simultaneously address the formation stabilization and formation tracking problems without switching. Moreover, the structure of the controller is relatively simple compared to the existing controllers in the literature, and thus, is practical and easy to implement. Simulations on a group of underactuated vehicles including nonholonomic mobile robots and surface vessels are presented to demonstrate the effectiveness of the proposed control scheme.

1. Introduction

1.1. Motivation

A mechanical system is underactuated if it has fewer number of independent actuators than its degrees of freedom. Planar vehicle systems with first-order or second-order nonintegrable constraints are typical examples of this kind. Motion control of planar underactuated vehicle systems has received much consideration in the last two decades due to its intrinsic nonlinear properties and practical applications [1–3]. As a consequence of the underactuation, planar underactuated vehicles with zero gravitational and buoyant field do not meet the Brockett's necessary condition [4] and thus cannot be asymptotically stabilized by continuous pure-state feedback [5]. To circumvent this difficulty, smooth time-varying feedback [6], discontinuous time-invariant feedback [7], and hybrid feedback [8] have been developed for the stabilization problem. Another obstruction is that there exists no universal continuous controller (even time-varying) that can track an arbitrary feasible trajectory [9]. In contrast with the case of fully-actuated systems, set-point stabilization cannot be considered as a special case of trajectory tracking. Thus, for planar underactuated systems, trajectory tracking and set-point stabilization usually are studied as two separated problems.

Controlled collective behaviors of multi-vehicle systems are of particular interest in recent years due to their potential applications ranging from industry to military [10]. The distributed formation control problem, which can be considered as classical trajectory tracking or stabilization control problem extended to the multi-agent systems, is one of the most actively studied topics within the field of control engineering. The distributed formation control consists of making all the agents form a predefined geometrical configuration through *local* interactions with or without a group reference [11]. In other words, each follower uses only *local* information/measurements to achieve a *global* formation task. It should be pointed out that in order to make formation relative to global coordinates, at least one agent in the network (e.g., the leader) needs to know its global positioning while the rest of the formation does *not*. On the other hand, in case of a leaderless network making a formation irrespective of the global coordinates, no agent needs to know their global coordinates and only relative measurements are required to guarantee the formation. Among various control schemes, the leader–follower strategy is of particular significance in many applications due to its simplicity and scalability [12]. Within this framework, many research articles have addressed the problem of planar underactuated vehicle formation control [13–15].

Because of the underactuation constraint, the formation stabilization and the formation tracking problems usually are studied as two distinct problems in the literature. Consequently, all

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agents must know the control problem in advance, and switch between the two different types of controllers, i.e. formation stabilization controller and formation tracking controller. However, switching between controllers may be impractical when the vehicles operate in a fully autonomous mode [16]. Furthermore, in a distributed network, it is only the group leader that knows the group reference and the control objective, while no prior information on the group reference trajectory is available to all other agents [17]. Therefore, it is more practical if the two problems can be solved using a single control architecture.

1.2. Related works

The problem of *simultaneous stabilization and tracking* refers to finding a single control law which can solve both stabilization and tracking problems simultaneously without changing the controller structure [18]. This problem was first addressed for nonholonomic mobile robots in [19] using a saturation feedback and backstepping technique. Then, an output feedback controller [20] and an adaptive controller [21] were proposed using the same backstepping idea as in [19], where a sinusoid signal is introduced in the angular velocity virtual control to handle the set-point stabilization. In [16], a controller was proposed for simultaneous stabilization and tracking of a mobile robot by introducing a time-varying signal, where the signal facilitates the conversion from a stabilization controller to a tracking controller adaptively and smoothly. Using the same saturation feedback idea in [19], an input-restricted robust controller was proposed in [22] to handle the parameter uncertainty and input constraints for mobile robots. In [23], a uniform δ-persistently exciting (uδ-PE) controller was proposed for nonholonomic mobile robots and *uniform* global asymptotic stability (UGAS) for the origin of the closed-loop system was established for the first time in the literature.

Unlike nonholonomic mobile robots, a surface vessel with only two available controls is under a nonintegrable second order nonholonomic constraint and is not transformable into a chained system. Thus, the controllers designed specifically for mobile robots [16,19–23] cannot be extended to underactuated surface vessels directly. The simultaneous stabilization and tracking problem for underactuated surface vessels was first addressed in [24] using a high-gain feedback, which achieves tracking and stabilization in the sense of global uniform ultimate boundedness (GUUB). Based on Lyapunov's direct method and backstepping technique, a time-varying controller was developed in [25] which guarantees the global asymptotic convergence of the stabilization and tracking errors to the origin. Then, an output-feedback controller [26] was designed using the same backstepping idea as in [25]. However, the designs in [25,26] are quite complicated, computationally demanding, and are heavily dependent on particular ship dynamics with linear hydrodynamic damping, which makes those approaches less practical. It is noted that while there are many approaches to design controllers for different kinds of planar underactuated vehicles, they are heavily dependent on the particular structures of the vehicles. In the authors' previous work [3], a trajectory tracking control framework was proposed for the generic planar underactuated vehicles that can be applied to various forms of planar vehicles.

For multi-agent systems, the problem of *formation stabilization and tracking* is a natural extension of the classical simultaneous stabilization and tracking problem. Using a distributed estimation strategy, the problem was solved in [27] for nonholonomic mobile robot networks in the sense of GUUB. In [15], a $u\delta$-PE formation stabilization and tracking controller was proposed for mobile robot networks using cascade system theory and Lyapunov's direct method (see also [28]). Based on generalized canonical transformations, a passivity-based formation stabilization and tracking controller was proposed for mobile robots in [29]. In practical applications, the vehicles in the network may be of different types. Even the same type of vehicles may have different dynamics and parameters due to the various sizes and loads. Thus, it is more practical if a group of planar vehicles can cooperate with each other regardless of the parameters or even structures of their dynamic models. To the best of authors' knowledge, formation stabilization and tracking control for *heterogeneous* planar underactuated vehicle networks is still an open problem.

1.3. Contributions of this paper

In this paper, we develop a new leader–follower formation control framework for a class of heterogeneous planar underactuated vehicle networks. Specifically,

(1) We solve the simultaneous *formation stabilization and tracking* problem for planar underactuated vehicle systems using a single smooth time-varying control architecture. The control design is developed based on uδ-PE, and guarantees UGAS for the origin of the closed-loop system.
(2) We do not assume any particular structure of the internal dynamics of each vehicle but rather use a generic Euler–Lagrangian (EL) model, and the vehicles are allowed to have identical or non-identical dynamics. In other words, the formation is designed for *heterogeneous* planar underactuated vehicle networks, which includes vehicles of different dynamic nature with different numbers of states (e.g., ground vehicles and surface vessels).
(3) The proposed control law requires only neighbor-to-neighbor information exchange, and does not require any global position measurements of the followers. Furthermore, the structure of the controller is relatively simple compared to the existing controllers in the literature.

Compared with existing results in the literature and in contrast to existing controllers in [15,27,28], which are applicable only to mobile robots, the approach proposed in this article can be applied to networks of *heterogeneous* vehicles that include not only vehicles with different parameters, but also vehicles of distinct dynamic nature where even a number of dynamic states for vehicles can differ. In contrast to our previous formation controller in [29], the approach proposed in this article requires only neighbor-to-neighbor information exchange, and does not require any global position measurements of the followers.

1.4. Outline

The rest of paper is organized as follows. Preliminaries and problem formulation are given in Section 2. Section 3 presents the main results, which are the leader–follower formation stabilization and tracking design and the stability analysis. Applications and numerical simulations are presented in Section 4. Finally, the concluding remarks are provided in Section 5.

2. Preliminaries and problem formulation

2.1. Notations

Let $\mathbb{R}^n$ represent the n-dimensional Euclidean space; $\mathbb{R}_{\geq 0}$ the set of all non-negative real numbers; $|\cdot|$ the Euclidean norm of vectors in $\mathbb{R}^n$. For any constant $\rho > 0$, we use the notations $\mathcal{B}_\rho := \{x \in \mathbb{R}^n : |x| < \rho\}$ and $\bar{\mathcal{B}}_\rho := \{x \in \mathbb{R}^n : |x| \leq \rho\}$ to denote open and closed ball of radius ρ, respectively. For a full-rank matrix $G \in \mathbb{R}^{n\times m}$ with $n > m$, we denote the generalized inverse as $G^\dagger := [G^\top G]^{-1}G^\top$, and define $\mathtt{sym}(G) := \frac{1}{2}(G+G^\top)$. Throughout this paper, we omit the arguments of functions when they are clear from the context. For multi-agent systems considered in this paper, we use the bold and non-italicized subscript $\mathbf{i}$ to denote the index of an agent.

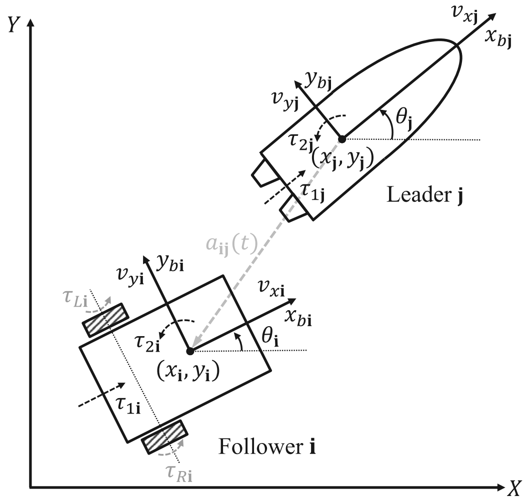


Fig. 1. Top view of the leader–follower formation of heterogeneous planar underactuated vehicles **i** and **j**, where the leader is an underactuated ship and the follower is a nonholonomic mobile robot.

2.2. Model of planar underactuated vehicles

Without loss of generality, a planar underactuated vehicle can be modeled as a 3-DOF planar rigid body with only two independent control inputs. The motion of a single vehicle **i** in the network is described by assigning a body-fixed reference frame $\{x_{b\mathbf{i}}y_{b\mathbf{i}}\}$ to its center of mass located at $(x_{\mathbf{i}}, y_{\mathbf{i}})$ and its orientation angle $\theta_{\mathbf{i}}$ with respect to a fixed inertial reference frame $\{XY\}$, as shown in Fig. 1. The mathematical model of the planar underactuated vehicle **i** can be written in the EL form [3,30]

$$\dot{q}_{\mathbf{i}} = J(q_{\mathbf{i}})v_{\mathbf{i}}, \tag{1a}$$

$$M_{\mathbf{i}}\dot{v}_{\mathbf{i}} + C_{\mathbf{i}}(v_{\mathbf{i}})v_{\mathbf{i}} + D_{\mathbf{i}}(v_{\mathbf{i}})v_{\mathbf{i}} = G_{\mathbf{i}}\tau_{\mathbf{i}}, \tag{1b}$$

where $q_{\mathbf{i}} = [x_{\mathbf{i}}, y_{\mathbf{i}}, \theta_{\mathbf{i}}]^\top$ is the configuration of the **i**th vehicle; $v_{\mathbf{i}} = [v_{x\mathbf{i}}, v_{y\mathbf{i}}, \omega_{\mathbf{i}}]^\top$ is the generalized velocity vector consisting of the velocity of the center of mass $(v_{x\mathbf{i}}, v_{y\mathbf{i}})$ in the body-fixed frame $\{x_{b\mathbf{i}}y_{b\mathbf{i}}\}$ and its angular velocity $\omega_{\mathbf{i}}$; $\tau_{\mathbf{i}} = [\tau_{1\mathbf{i}}, \tau_{2\mathbf{i}}]^\top$ is the control input vector; $J(q_{\mathbf{i}})$ is the orthogonal kinematic transformation matrix given by

$$J(q_{\mathbf{i}}) = \begin{bmatrix} \cos(\theta_{\mathbf{i}}) & -\sin(\theta_{\mathbf{i}}) & 0 \\ \sin(\theta_{\mathbf{i}}) & \cos(\theta_{\mathbf{i}}) & 0 \\ 0 & 0 & 1 \end{bmatrix}; \tag{2}$$

$M_{\mathbf{i}}$ is the inertia matrix; $C_{\mathbf{i}}(v_{\mathbf{i}})$ is the Coriolis and centrifugal matrix; $D_{\mathbf{i}}(v_{\mathbf{i}})$ is the damping matrix; and $G_{\mathbf{i}}$ is the input matrix. All matrices above are assumed to be in appropriate dimensions. Three well-known properties associated with the EL system (1a), (1b) are as follows.

Property 1 ([31])**.** *For a single rigid body, the inertia matrix $M_{\mathbf{i}}$ is constant, symmetric and positive definite, and the Coriolis and centrifugal matrix $C_{\mathbf{i}}(v_{\mathbf{i}})$ is skew-symmetric.*

Property 2 ([31])**.** *The damping matrix $D_{\mathbf{i}}(v_{\mathbf{i}})$ is symmetric and positive semi-definite.*

Property 3 ([30])**.** *For the system (1a), (1b), the differential equation*

$$M_{\mathbf{i}}\dot{s}_{\mathbf{i}} + C_{\mathbf{i}}(v_{\mathbf{i}})s_{\mathbf{i}} + D_{\mathbf{i}}(v_{\mathbf{i}})s_{\mathbf{i}} = G_{\mathbf{i}}\tau_{\mathbf{i}} \tag{3}$$

defines an input–output mapping $\tau_{\mathbf{i}} \mapsto y_{\mathbf{i}} := G_{\mathbf{i}}^\top s_{\mathbf{i}}$, which is passive with the storage function $E_K := \frac{1}{2}s_{\mathbf{i}}^\top M_{\mathbf{i}}s_{\mathbf{i}}$. Furthermore, if $D_{\mathbf{i}}(\cdot)$ is positive definite, then the mapping $\tau_{\mathbf{i}} \mapsto y_{\mathbf{i}}$ is output strictly passive.

Without loss generality, we make the following assumption.

Assumption 1. (i.) For each vehicle **i**, assume that the inertia matrix $M_{\mathbf{i}}$ is diagonal, i.e., $M_{\mathbf{i}} = \mathrm{diag}(m_{11,\mathbf{i}}, m_{22,\mathbf{i}}, m_{33,\mathbf{i}})$. (ii.) Assume that the surge force and the yaw torque are two independent control inputs. That is, the input matrix $G_{\mathbf{i}}$ may be written as[1]

$$G_{\mathbf{i}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \tag{4}$$

which implies that the underactuation is in the sway direction, i.e., $v_{y\mathbf{i}}$-equation. (iii.) Assume that for each vehicle, the damping force in the sway direction satisfies $[D_{\mathbf{i}}(v_{\mathbf{i}})]_{(2,2)} > 0$ for all $v_{y\mathbf{i}} \neq 0$, and $v_{y\mathbf{i}}/[D_{\mathbf{i}}(v_{\mathbf{i}})]_{(2,2)} \to 0$ as $v_{y\mathbf{i}} \to 0$, where $[D_{\mathbf{i}}(v_{\mathbf{i}})]_{(2,2)}$ denotes the $(2, 2)$-element of $D_{\mathbf{i}}(v_{\mathbf{i}})$.

Remark 1. The EL system (1a), (1b) with Assumption 1 can model a wide class of planar underactuated vehicles in practical applications such as nonholonomic mobile robots [1,32], underactuated ships [2,3], underwater vehicles [26], etc. The assumption of damping force for the sway velocity is a mild one and has been adopted in the literature on the topic of underactuated ships [14]. Note that this assumption is automatically satisfied in case of a linear damping model.

2.3. Graph theory

For formation control of planar underactuated vehicle networks, we use graph theory to define the communication interaction among the vehicles. Consider a network of $N + 1$ heterogeneous planar underactuated vehicles, where the vehicles are numbered $\mathbf{i} = 0, 1, \ldots, N$ with $\mathbf{0}$ representing the real group leader and $\mathbf{1}, \ldots, \mathbf{N}$ the follower agents. The network topology of the vehicles is defined by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{\mathbf{0}, \mathbf{1}, \ldots, \mathbf{N}\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represent its sets of vertices and edges, respectively. The set of neighboring nodes with edges connected to node **i** is denoted by $\mathcal{N}_{\mathbf{i}} = \{\mathbf{j} \in \mathcal{V} : (\mathbf{i}, \mathbf{j}) \in \mathcal{E}\}$. The edges represent communication between the nodes such that (follower) node **i** obtains information from (leader) node **j** for feedback control purposes, if $\mathbf{j} \in \mathcal{N}_{\mathbf{i}}$, as shown in Fig. 1. The constant weighted adjacency matrix $\mathcal{A} = [a_{\mathbf{ij}}]$ associated with $\mathcal{G}$ is defined in accordance with the rule that $a_{\mathbf{ij}} > 0$ in the case that $\mathbf{j} \in \mathcal{N}_{\mathbf{i}}$ and $a_{\mathbf{ij}} = 0$ otherwise. The physical meaning of the weighting coefficients $a_{\mathbf{ij}} > 0$ represents the different levels of importance of the agent neighbors' information states. For the group leader, we have $a_{\mathbf{0j}} \equiv 0$ for all $\mathbf{j} \in \mathcal{V}$, which implies that the leader **0** has no neighbors in the network. We also assume that $a_{\mathbf{ii}} = 0$ for all $\mathbf{i} \in \mathcal{V}$. The communication graph $\mathcal{G}$ is assumed to contain a directed spanning tree [33], which implies that there exists at least one directed path consisting of communication edges from the group leader **0** to each agent in the network. That is the information from the leader passes down to an agent in the network which, in turn, sends its own information to the neighboring agents and so on. For more details on algebraic graph theory, see [10,33].

[1] $G_{\mathbf{i}}$ may be obtained in form (4) via a pre-input transformation. For instance, the actual control inputs for a mobile robot are torques $\tau_{L\mathbf{i}}$ and $\tau_{R\mathbf{i}}$ applied to each wheel, as shown in Fig. 1. However, one can easily obtain the input matrix of the form (4) via a linear transformation of the actual inputs. See [32] for more details.

2.4. Problem formulation

We assume that the reference trajectory of the group leader is feasible and is generated by the following virtual vehicle

$$\dot{q}_{\mathrm{d}} = J(q_{\mathrm{d}})v_{\mathrm{d}}, \tag{5}$$

where $q_{\mathrm{d}} = [x_{\mathrm{d}}, y_{\mathrm{d}}, \theta_{\mathrm{d}}]^\top$ denotes the position and orientation of the virtual vehicle, and $v_{\mathrm{d}} = [v_{x\mathrm{d}}, v_{y\mathrm{d}}, \omega_{\mathrm{d}}]^\top$ denotes the linear and angular velocities of the virtual vehicle. We make the following assumption on the reference trajectory of the group leader.

Assumption 2. The reference trajectory $(q_{\mathrm{d}}(t), v_{\mathrm{d}}(t))$ is only available to the group leader **0**. Furthermore, the reference velocity $v_{\mathrm{d}}(\cdot)$ is continuously differentiable and bounded with bounded first derivative. Moreover, one of the following conditions holds.

(A1) There exist T and $\mu_1 > 0$ such that

$$\int_t^{t+T} \omega_{\mathrm{d}}(\tau)^2 \mathrm{d}\tau \geq \mu_1, \ \forall t \geq 0. \tag{6}$$

(A2) There exist $\mu_2 > 0$ such that

$$\int_0^{\infty} |\omega_{\mathrm{d}}(\tau)| \mathrm{d}\tau \leq \mu_2. \tag{7}$$

The objective of *formation stabilization and tracking* is to design a distributed controller for each agent such that it coordinates its motion relative to one or more of its neighbors, and the network asymptotically converges to a predefined geometric pattern under Assumption 2. The geometric pattern of vehicle network in terms of planar configuration is defined by a set of constant offset vectors $\{d_{\mathbf{ij}} := (d_{\mathbf{ij}}^x, d_{\mathbf{ij}}^y, d_{\mathbf{ij}}^\theta) \in \mathbb{R}^3 : \mathbf{i}, \mathbf{j} \in \mathcal{V}, \mathbf{i} \neq \mathbf{j}\}$. To be more specific, under Assumption 2, we will design a controller $(\tau_{1\mathbf{i}}, \tau_{2\mathbf{i}})$ for each agent without global position measurements such that: (i.) all states in the closed-loop system are uniformly bounded; (ii.) all the vehicles in the network maintain a prescribed formation in the sense that for all $\mathbf{i} \in \mathcal{V}$

$$\lim_{t\to\infty} \sum_{\mathbf{j}\in\mathcal{N}_\mathbf{i}} \left| q_\mathbf{i}(t) - q_\mathbf{j}(t) - d_{\mathbf{ij}} \right| = 0. \tag{8}$$

In this paper, we choose constant offset vectors $(d_{\mathbf{ij}}^x, d_{\mathbf{ij}}^y, d_{\mathbf{ij}}^\theta)$ for simplicity of exposition and the results can be extended to address time-varying formation geometry.

Remark 2. In Assumption 2, *A1* implies that $\omega_{\mathrm{d}}(\cdot)$ is persistently exciting (PE), and *A2* implies that $\omega_{\mathrm{d}}(\cdot)$ belong to L_1-space, which implies that the reference angular velocity converge to zero sufficiently fast. Hence, formation tracking of trajectories such as a circle or a sinusoid function is included in *A1*, while formation stabilization or formation tracking of a straight line is included in *A2*. Note that the formation stabilization and tracking problem covers two important cases:

Case 1 (Formation Stabilization): If $v_{x\mathrm{d}}(t) = 0$, $v_{y\mathrm{d}}(t) = 0$, and $\omega_{\mathrm{d}}(t) = 0$ for all $t \geq 0$, then the formation stabilization and tracking problem is reduced to the formation stabilization problem.

Case 2 (Formation Tracking): If $\lim_{t\to\infty}[v_{x\mathrm{d}}^2(t) + \omega_{\mathrm{d}}^2(t)] \neq 0$, then the formation stabilization and tracking problem is reduced to the formation tracking control problem.

2.5. Technical lemmas

In this subsection, we review and develop some results needed for the main results of the paper.

2.5.1. Partial stability conditions for UGAS of interconnected systems

For basic definitions and the use of partial stability in the analysis of interconnected systems, the readers are referred to [34,35]. Consider the following time-varying interconnected system

$$\Sigma_1 : \dot{x}_1 = f_1(t, x_1, x_2), \ \ x_1(t_0) = x_{10}, \ \ t_0 \geq 0, \tag{9}$$

$$\Sigma_2 : \dot{x}_2 = f_2(t, x_1, x_2), \ \ x_2(t_0) = x_{20}, \tag{10}$$

where $x = (x_1, x_2) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$. We assume that the functions f_1, f_2 are continuous in their arguments, locally Lipschitz in (x_1, x_2), uniformly in t, and the origin $(x_1, x_2) = (0, 0)$ is an equilibrium point. For nonlinear time-varying system (9), (10) with partial stability, we give sufficient conditions to guarantee the UGAS of origin.

Theorem 1. *Suppose that f_2 is continuously differentiable. Then, the origin of the interconnected system (9), (10) is UGAS if the following conditions hold.*

(1) (Partial stability with respect to x_1) There exist a continuously differentiable function $V_1 : \mathbb{R} \times \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \to \mathbb{R}_{\geq 0}$, functions $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$, and a positive definite function $W_1 : \mathbb{R}^{n_1} \to \mathbb{R}$ such that

$$\alpha_1(|x_1|) \leq V_1(t, x_1, x_2) \leq \alpha_2(|x_1|), \tag{11}$$

$$\dot{V}_1(t, x_1, x_2) \leq -W_1(x_1), \tag{12}$$

for all $(t, x_1, x_2) \in \mathbb{R} \times \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$.

(2) (0-UGAS of Σ_2) There exist a continuously differentiable function $V_2 : \mathbb{R} \times \mathbb{R}^{n_2} \to \mathbb{R}_{\geq 0}$, functions $\alpha_3, \alpha_4 \in \mathcal{K}_\infty$, function $\alpha_5 \in \mathcal{K}$, and a positive definite function $W_2 : \mathbb{R}^{n_2} \to \mathbb{R}$ such that

$$\alpha_3(|x_2|) \leq V_2(t, x_2) \leq \alpha_4(|x_2|), \tag{13}$$

$$\frac{\partial V_2}{\partial t} + \frac{\partial V_2}{\partial x_2} f_2(t, 0, x_2) \leq -W_2(x_2), \tag{14}$$

$$\left| \frac{\partial V_2}{\partial x_2} \right| \leq \alpha_5(|x_2|), \tag{15}$$

for all $(t, x_2) \in \mathbb{R} \times \mathbb{R}^{n_2}$.

(3) ($|x_1|$ is small order of W_1) The function W_1 satisfies

$$\lim_{|x_1|\to\infty} \frac{|x_1|}{W_1(x_1)} = 0. \tag{16}$$

Proof. Along the trajectories of (9), (10), we have

$$\begin{aligned} \dot{V}_2 &\leq -W_2(x_2) + \left[\frac{\partial V_2}{\partial x_2} f_2(t, x_1, x_2) - \frac{\partial V_2}{\partial x_2} f_2(t, 0, x_2) \right] \\ &\leq -W_2(x_2) + \left| \frac{\partial V_2}{\partial x_2} \right| \left| \frac{\partial f_2}{\partial x_1} \right| |x_1|. \end{aligned}$$

Since V_2 is continuously differentiable and f_2 is continuous and Lipshitz, it follows that for each $r > 0$ there exist $c_1 > 0$ and $c_2 > 0$ such that $|\partial V_2/\partial x_2| \leq c_1$ and $|\partial f_2/\partial x_1| \leq c_2$ for all $t \geq 0$ and for all $(x_1, x_2) \in \bar{\mathcal{B}}_r$. Then, consider a Lyapunov candidate $V = \kappa V_1 + V_2$, where κ is a positive constant. Along the trajectories of (9), (10), we have

$$\dot{V}(t, x_1, x_2) \leq -\kappa W_1(x_1) \left[1 - \frac{c_1 c_2}{\kappa W_1(x_1)} |x_1| \right] - W_2(x_2). \tag{17}$$

It follows from (16), (17) that the system (9), (10) is uniformly globally bounded (UGB) by choosing κ sufficiently large. It follows from [35, Theorem 3.1] that the origin of system (9), (10) is uniformly asymptotically stable. Thus, there exists $\delta > 0$ such that $|x(t_0)| < \delta \Rightarrow |x(t, t_0, x(t_0))| \to 0$ as $t \to \infty$. The uniform global attractivity follows from the fact that κ can be chosen arbitrarily large such that the trajectory of (9), (10) with initial conditions starting in $\bar{\mathcal{B}}_r$ enters the domain of attraction $\mathcal{B}_\delta$ for any $r > 0$. □

2.5.2. Matrosov's theorem

Our main result also relies on Matrosov's theorem concerning the differential equation $\dot{x} = f(t,x)$ with an equilibrium point at the origin.

Definition 1 (*Non-zero Definiteness [36]*)**.** A continuous function $w : \mathbb{R}_{\geq 0} \times \bar{\mathcal{B}}_\rho \to \mathbb{R}$ is said to be non-zero definite on the set $M \subset \bar{\mathcal{B}}_\rho$ if for any pair of numbers δ and R such that $0 < \delta < R \leq \rho$ there exist positive numbers Δ and μ such that

$$\left.\begin{array}{l} |x| \in [\delta, R] \\ |x|_M < \Delta \\ t \geq 0 \end{array}\right\} \implies |w(t,x)| > \mu, \tag{18}$$

where $|x|_M := \inf_{z \in M} |x - z|$.

Theorem 2 (*Matrosov's Theorem [36]*)**.** *Suppose that there exist a continuous function $V^\star : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$, continuously differentiable functions $V : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}$ and $W : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}$, functions $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$, and for each $R > 0$, there exists $L > 0$ such that*

(a) W and f satisfy

$$\max\{|W(t,x)|, |f(t,x)|\} \leq L, \quad \forall (t,x) \in \mathbb{R}_{\geq 0} \times \bar{\mathcal{B}}_R; \tag{19}$$

(b) V is positive definite decrescent and $\dot{V}$ is negative semi-definite, i.e., for all $(t,x) \in \mathbb{R}_{\geq 0} \times \mathbb{R}^n$

$$\alpha_1(|x|) \leq V(t,x) \leq \alpha_2(|x|), \tag{20}$$

$$\dot{V}(t,x) \leq -V^\star(x) \leq 0; \tag{21}$$

(c) the function $\dot{W}(t,x)$ is non-zero definite on

$$M := \left\{x \in \bar{\mathcal{B}}_R : V^\star(x) = 0\right\}. \tag{22}$$

Then, the origin of $\dot{x} = f(t,x)$ is UGAS.

The proof of Theorem 2 is provided in [36].

3. Leader–follower formation control

3.1. Feasible reference trajectory generation

Due to the nonholonomic constraints of underactuated vehicles, the geometric pattern dictated by the formation $\{d_{\mathbf{ij}} \in \mathbb{R}^3 : \mathbf{i},\mathbf{j} \in \mathcal{V}\}$ cannot be assigned arbitrarily. More precisely, for each vehicle $\mathbf{i}$, given the desired *position* offset vectors $(d^x_{\mathbf{ij}}, d^y_{\mathbf{ij}})$ for all $\mathbf{j} \in \mathcal{N}_\mathbf{i}$, the feasible *orientation* offset $d^\theta_{\mathbf{ij}}$ must be determined based on the same nonholonomic constraints. For illustration, let us denote the position reference trajectory for the vessel $\mathbf{i}$ by

$$\bar{x}_\mathbf{i}(t) := \frac{1}{\sum_{\mathbf{j}\in\mathcal{N}_\mathbf{i}} a_{\mathbf{ij}}} \sum_{\mathbf{j}\in\mathcal{N}_\mathbf{i}} a_{\mathbf{ij}} \left[x_\mathbf{j}(t) + d^x_{\mathbf{ij}}\right], \tag{23}$$

$$\bar{y}_\mathbf{i}(t) := \frac{1}{\sum_{\mathbf{j}\in\mathcal{N}_\mathbf{i}} a_{\mathbf{ij}}} \sum_{\mathbf{j}\in\mathcal{N}_\mathbf{i}} a_{\mathbf{ij}} \left[y_\mathbf{j}(t) + d^y_{\mathbf{ij}}\right]. \tag{24}$$

Then, the feasible orientation trajectory $\bar{\theta}_\mathbf{i}(t)$ is forced to obey the same second-order nonholonomic constraint as the vehicle $\mathbf{i}$. That is, $\bar{\theta}_\mathbf{i}(t)$ is the solution of the underactuated equation in (1b) (i.e., the $v_{y\mathbf{i}}$-equation)

$$\dot{\bar{v}}_{y\mathbf{i}}(t) = f_{y\mathbf{i}}\left(\bar{v}_{x\mathbf{i}}(t), \bar{v}_{y\mathbf{i}}(t), \dot{\bar{\theta}}_\mathbf{i}(t)\right), \tag{25}$$

subjected to the initial condition $\bar{\theta}_\mathbf{i}(0) = \bar{\theta}_{\mathbf{i},0}$ [3], where $f_{y\mathbf{i}}$ is the right hand side of the second equation in

$$\dot{\bar{v}}_\mathbf{i} = -M_\mathbf{i}^{-1}C_\mathbf{i}(\bar{v}_\mathbf{i})\bar{v}_\mathbf{i} - M_\mathbf{i}^{-1}D_\mathbf{i}(\bar{v}_\mathbf{i})\bar{v}_\mathbf{i} + M_\mathbf{i}^{-1}G_\mathbf{i}\tau_\mathbf{i}. \tag{26}$$

Let us denote $\bar{q}_\mathbf{i}(t) := [\bar{x}_\mathbf{i}(t), \bar{y}_\mathbf{i}(t), \bar{\theta}_\mathbf{i}(t)]^\top$, and $\bar{v}_\mathbf{i}(t) := [\bar{v}_{x\mathbf{i}}(t), \bar{v}_{y\mathbf{i}}(t), \dot{\bar{\theta}}_\mathbf{i}(t)]^\top$. Note that components $\bar{v}_{x\mathbf{i}}$ and $\bar{v}_{y\mathbf{i}}$ can be expressed in terms of $\bar{\theta}_\mathbf{i}$ and $\dot{\bar{\theta}}_\mathbf{i}$ using Eq. (1a). As a result, (25) is a first-order nonlinear ordinary differential equation with respect to $\bar{\theta}_\mathbf{i}$. We integrate this equation *numerically* in real time to obtain the feasible orientation trajectory $\bar{\theta}_\mathbf{i}(t)$ as a function of time. Finally, the feasible orientation offset $d^\theta_{\mathbf{ij}}$ is selected as $d^\theta_{\mathbf{ij}} = \bar{\theta}_\mathbf{i}(t) - \theta_\mathbf{j}(t)$.

Remark 3. It is noted that we only use the time derivative of (23), (24) in the feasible reference trajectory generation, and the global position measurements $(x_\mathbf{j}(t), y_\mathbf{j}(t))$ are not required. Also, we will use the signal $q_\mathbf{i}(t) - \bar{q}_\mathbf{i}(t)$ for feedback purpose, and only the *relative* configuration measurements $q_\mathbf{i}(t) - q_\mathbf{j}(t)$ are required. See an example of reference trajectory generation for underactuated surface vessels in [37]. In the case of formation control for *homogeneous* vehicle networks, $d^\theta_{\mathbf{ij}}$ can be chosen simply as 0. Note, however, that in order to guarantee a formation relative to *global* coordinates, the group leader $\mathbf{0}$ needs to know its global positioning while the rest of the network does *not* need to.

3.2. Formation control design

For the set-point stabilization problem of fully-actuated EL systems without kinematics equations

$$M_\mathbf{i}\ddot{q}_\mathbf{i} + C_\mathbf{i}(\dot{q}_\mathbf{i})\dot{q}_\mathbf{i} + D_\mathbf{i}(\dot{q}_\mathbf{i})\dot{q}_\mathbf{i} = \tau_\mathbf{i}, \tag{27}$$

a fundamental result is achieving global asymptotic stabilization via energy shaping plus damping injection, where the controller always has a simple proportional–derivative (PD) form [30], i.e.,

$$\tau_\mathbf{i} = -k_{p\mathbf{i}}(q_\mathbf{i} - \bar{q}_\mathbf{i}) - k_{d\mathbf{i}}\dot{q}_\mathbf{i}, \tag{28}$$

where $k_{p\mathbf{i}}, k_{d\mathbf{i}}$ are positive control gains. For the tracking control problem of fully-actuated EL systems (27), the PD+ controller originally introduced in [38] is a natural extension of the PD control law (28) and is given by

$$\tau_\mathbf{i} = M\ddot{\bar{q}}_\mathbf{i} + C(\dot{q}_\mathbf{i})\dot{\bar{q}}_\mathbf{i} + D(\dot{q}_\mathbf{i})\dot{\bar{q}}_\mathbf{i} - k_{p\mathbf{i}}(q_\mathbf{i} - \bar{q}_\mathbf{i}) - k_{d\mathbf{i}}(\dot{q}_\mathbf{i} - \dot{\bar{q}}_\mathbf{i}). \tag{29}$$

The PD+ controller was proved in [38] to achieve global asymptotic tracking using Matrosov's theorem. It is noted that the PD+ controller (29) reduces to the PD controller (28) when the reference velocity tends to zero. We will use a similar passivity-based technique in the simultaneous formation stabilization and tracking control design.

For the leader–follower tracking problem, we usually consider the problem for the follower $\mathbf{i}$ as tracking a reference leader similar to [1,3,15,28,29,37]. The basic idea is to calculate the dynamics of the tracking error $(q_\mathbf{i} - \bar{q}_\mathbf{i}, v_\mathbf{i} - \bar{v}_\mathbf{i})$, and try to stabilize this error system. However, the error system often becomes very complex. Thus, instead of using $\bar{v}_\mathbf{i}$, we define the new reference velocity in the body-fixed frame $\{x_{b\mathbf{i}}y_{b\mathbf{i}}\}$ as $\hat{v}_\mathbf{i} := J(q_\mathbf{i})^\top\dot{\bar{q}}_\mathbf{i}$. Correspondingly, for agent $\mathbf{i}$, the error vectors in the body-fixed frame $\{x_{b\mathbf{i}}y_{b\mathbf{i}}\}$ are defined as $\tilde{q}^b_\mathbf{i} = [\tilde{x}^b_\mathbf{i}, \tilde{y}^b_\mathbf{i}, \tilde{\theta}_\mathbf{i}]^\top := J(q_\mathbf{i})^\top(q_\mathbf{i} - \bar{q}_\mathbf{i})$, and $\tilde{v}_\mathbf{i} = [\tilde{v}_{x\mathbf{i}}, \tilde{v}_{y\mathbf{i}}, \tilde{\omega}_\mathbf{i}]^\top := (v_\mathbf{i} - \hat{v}_\mathbf{i})$. Clearly, since $J(q_\mathbf{i})$ is invertible, stabilization of $(\tilde{q}^b_\mathbf{i}, \tilde{v}_\mathbf{i})$ implies that $q_\mathbf{i}(t) \to \bar{q}_\mathbf{i}(t)$ and $\dot{q}_\mathbf{i}(t) \to \dot{\bar{q}}_\mathbf{i}(t)$ as $t \to \infty$ which solves formation control problem (8). Let us consider the following modified PD+ controller

$$\tau_\mathbf{i} = G_\mathbf{i}^\dagger\left[M\dot{\hat{v}}_\mathbf{i} + C(v_\mathbf{i})\hat{v}_\mathbf{i} + D(v_\mathbf{i})\hat{v}_\mathbf{i} - K_{p\mathbf{i}}\tilde{q}^b_\mathbf{i} - K_{d\mathbf{i}}\tilde{v}_\mathbf{i} + u_\mathbf{i}\right], \tag{30}$$

where $K_{p\mathbf{i}} > 0$ and $K_{d\mathbf{i}} > 0$ are constant, diagonal control gain matrices; $u_\mathbf{i}$ is a new control input which will be designed later. We have the following result.

Proposition 1. *Consider the planar underactuated vehicle (1a), (1b) satisfying Assumption 1. Then, under the modified PD+ control law (30) with $u_\mathbf{i} \equiv 0$, the origin for the $(\tilde{x}^b_\mathbf{i}, \tilde{\theta}_\mathbf{i}, \tilde{v}_{x\mathbf{i}}, \tilde{v}_{y\mathbf{i}}, \tilde{\omega}_\mathbf{i})$-subsystem is UGAS, and the solutions of the closed-loop system are UGB.*

Proof. Consider the function

$$V_{\mathbf{i}}(\tilde{q}_{\mathbf{i}},\tilde{v}_{\mathbf{i}})=\frac{1}{2}\left[\tilde{v}_{\mathbf{i}}^{\top}\left(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger}\right)M_{\mathbf{i}}\tilde{v}_{\mathbf{i}}+(\tilde{q}_{\mathbf{i}}^{b})^{\top}\left(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger}\right)K_{p\mathbf{i}}\tilde{q}_{\mathbf{i}}^{b}\right],$$

which is positive definite with respect to the error vector $(\tilde{x}_{\mathbf{i}}^{b},\tilde{\theta}_{\mathbf{i}},\tilde{v}_{x\mathbf{i}},\tilde{\omega}_{\mathbf{i}})$. Taking time derivative along the trajectories of the closed-loop system, we have

$$\begin{aligned}\dot{V}_{\mathbf{i}}=&\tilde{v}_{\mathbf{i}}^{\top}\left(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger}\right)M_{\mathbf{i}}\dot{\tilde{v}}_{\mathbf{i}}+(\dot{\tilde{q}}_{\mathbf{i}}^{b})^{\top}\left(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger}\right)K_{p\mathbf{i}}\tilde{q}_{\mathbf{i}}^{b}\\=&\tilde{v}_{\mathbf{i}}^{\top}\left(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger}\right)\left[G_{\mathbf{i}}\tau_{\mathbf{i}}-C_{\mathbf{i}}(v_{\mathbf{i}})v_{\mathbf{i}}-D_{\mathbf{i}}(v_{\mathbf{i}})v_{\mathbf{i}}-M_{\mathbf{i}}\dot{\hat{v}}_{\mathbf{i}}\right]\\&+(\dot{\tilde{q}}_{\mathbf{i}}^{b})^{\top}\left(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger}\right)K_{p\mathbf{i}}\tilde{q}_{\mathbf{i}}^{b}\\=&\tilde{v}_{\mathbf{i}}^{\top}\left(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger}\right)\left[C(v_{\mathbf{i}})\hat{v}_{\mathbf{i}}+D(v_{\mathbf{i}})\hat{v}_{\mathbf{i}}-K_{p\mathbf{i}}\tilde{q}_{\mathbf{i}}^{b}-K_{d\mathbf{i}}\tilde{v}_{\mathbf{i}}+u_{\mathbf{i}}\right.\\&-\left.C_{\mathbf{i}}(v_{\mathbf{i}})v_{\mathbf{i}}-D_{\mathbf{i}}(v_{\mathbf{i}})v_{\mathbf{i}}\right]+(\dot{\tilde{q}}_{\mathbf{i}}^{b})^{\top}\left(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger}\right)K_{p\mathbf{i}}\tilde{q}_{\mathbf{i}}^{b}\\=&-\tilde{v}_{\mathbf{i}}^{\top}\mathrm{sym}\left\{\left(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger}\right)[D_{\mathbf{i}}(v_{\mathbf{i}})+K_{d\mathbf{i}}]\right\}\tilde{v}_{\mathbf{i}}+\tilde{v}_{\mathbf{i}}^{\top}\left(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger}\right)u_{\mathbf{i}}\\\le&\left[\left(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger}\right)\tilde{v}_{\mathbf{i}}\right]^{\top}u_{\mathbf{i}},\end{aligned}\tag{31}$$

where the third equality is due to the fact that $G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger}$ is idempotent, the fourth one is due to Property 1, and the last inequality is due to Property 2 and $K_{d\mathbf{i}}>0$. It is clear that the input–output mapping $u_{\mathbf{i}}\mapsto(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger})\tilde{v}_{\mathbf{i}}$ is passive. Consequently, if $u_{\mathbf{i}}\equiv 0$, we have $(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger})\tilde{v}_{\mathbf{i}}\in L_2$, and the origin for the $(\tilde{x}_{\mathbf{i}}^{b},\tilde{\theta}_{\mathbf{i}},\tilde{v}_{x\mathbf{i}},\tilde{\omega}_{\mathbf{i}})$-subsystem is UGS. It also follows from LaSalle–Yoshizawa theorem that $(\tilde{v}_{x\mathbf{i}},\tilde{\omega}_{\mathbf{i}})\to 0$ as $t\to\infty$. If we consider $\tilde{v}_{y\mathbf{i}}(t)$ as a time-varying signal, then the origin of the $(\tilde{v}_{x\mathbf{i}},\tilde{\omega}_{\mathbf{i}})$-subsystem is uniformly globally exponentially stable. Then, the $(\tilde{v}_{x\mathbf{i}},\tilde{v}_{y\mathbf{i}},\tilde{\omega}_{\mathbf{i}})$-subsystem is globally exponential stable with respect to $(\tilde{v}_{x\mathbf{i}},\tilde{\omega}_{\mathbf{i}})$ uniformly in $\tilde{v}_{y\mathbf{i}}(0)$ (i.e., partial stability with respect to $(\tilde{v}_{x\mathbf{i}},\tilde{\omega}_{\mathbf{i}})$). It also follows from Assumption 1 item (iii.) that the origin of $\tilde{v}_{y\mathbf{i}}$-dynamics is UGAS when $(\tilde{v}_{x\mathbf{i}},\tilde{\omega}_{\mathbf{i}})\equiv(0,0)$ (i.e., 0-UGAS of $\tilde{v}_{y\mathbf{i}}$-subsystem). Therefore, we conclude that the $(\tilde{v}_{x\mathbf{i}},\tilde{v}_{y\mathbf{i}},\tilde{\omega}_{\mathbf{i}})$-subsystem is UGAS according to Theorem 1. Moreover, the condition $v_{y\mathbf{i}}/[D_{\mathbf{i}}(v_{\mathbf{i}})]_{(2,2)}\to 0$ as $v_{y\mathbf{i}}\to 0$ implies that $\tilde{v}_{y\mathbf{i}}\in L_1$ and $\tilde{y}_{\mathbf{i}}^{b}\in L_\infty$. Thus, we conclude that the solutions of the closed-loop system are UGB.

Next, consider the auxiliary function $W_{\mathbf{i}}=(\tilde{q}_{\mathbf{i}}^{b})^{\top}(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger})M_{\mathbf{i}}\tilde{v}_{\mathbf{i}}$ for the $(\tilde{x}_{\mathbf{i}}^{b},\tilde{\theta}_{\mathbf{i}},\tilde{v}_{x\mathbf{i}},\tilde{\omega}_{\mathbf{i}})$-subsystem. Taking time derivative of $W_{\mathbf{i}}$ along trajectories of the closed-loop system, we have

$$\dot{W}_{\mathbf{i}}=(\dot{\tilde{q}}_{\mathbf{i}}^{b})^{\top}(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger})M_{\mathbf{i}}\tilde{v}_{\mathbf{i}}+(\tilde{q}_{\mathbf{i}}^{b})^{\top}(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger})M_{\mathbf{i}}\dot{\tilde{v}}_{\mathbf{i}}.$$

Then, evaluating $\dot{W}$ on the set $\mathcal{M}:=\{\tilde{v}_{\mathbf{i}}=0\}$ yields

$$\begin{aligned}\dot{W}_{\mathbf{i}}|_{\mathcal{M}}=&-(\tilde{q}_{\mathbf{i}}^{b})^{\top}\mathrm{sym}\left\{\left(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger}\right)[C_{\mathbf{i}}(v_{\mathbf{i}})+D_{\mathbf{i}}(v_{\mathbf{i}})+K_{d\mathbf{i}}]\right\}\tilde{v}_{\mathbf{i}}\\&-(\tilde{q}_{\mathbf{i}}^{b})^{\top}(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger})K_{p\mathbf{i}}\tilde{q}_{\mathbf{i}}^{b}\\=&-(\tilde{q}_{\mathbf{i}}^{b})^{\top}(G_{\mathbf{i}}G_{\mathbf{i}}^{\dagger})K_{p\mathbf{i}}\tilde{q}_{\mathbf{i}}^{b}\le 0.\end{aligned}$$

Thus, $\dot{W}_{\mathbf{i}}$ is non-zero definite on the set $\mathcal{M}$. It follows from the Matrosov's Theorem 2 that the origin for the $(\tilde{x}_{\mathbf{i}}^{b},\tilde{\theta}_{\mathbf{i}},\tilde{v}_{x\mathbf{i}},\tilde{\omega}_{\mathbf{i}})$-subsystem is UGAS. Therefore, we conclude that the origin of the $(\tilde{x}_{\mathbf{i}}^{b},\tilde{\theta}_{\mathbf{i}},\tilde{v}_{x\mathbf{i}},\tilde{v}_{y\mathbf{i}},\tilde{\omega}_{\mathbf{i}})$-subsystem is UGAS by considering $\tilde{y}_{\mathbf{i}}^{b}(t)$ as a bounded time-varying signal. □

Under the modified passivity-based PD+ controller (30) with $u_{\mathbf{i}}\equiv 0$, the velocity error vector $\tilde{v}_{\mathbf{i}}(t)\to 0$, and the position error in the body-fixed frame $(\tilde{x}_{\mathbf{i}}^{b}(t),\tilde{\theta}_{\mathbf{i}}(t))\to 0$ as $t\to\infty$. However, due to the underactuation, the position error $\tilde{y}_{\mathbf{i}}^{b}(t)$ may converge only to a constant which is not necessarily zero. Denote the position error in the global frame by $(\tilde{x}_{\mathbf{i}},\tilde{y}_{\mathbf{i}}):=(x_{\mathbf{i}}-\bar{x}_{\mathbf{i}},y_{\mathbf{i}}-\bar{y}_{\mathbf{i}})$. Although

$$\tilde{x}_{\mathbf{i}}^{b}(t)=\begin{bmatrix}\cos(\theta_{\mathbf{i}}) & \sin(\theta_{\mathbf{i}})\end{bmatrix}\begin{bmatrix}\tilde{x}_{\mathbf{i}}(t) & \tilde{y}_{\mathbf{i}}(t)\end{bmatrix}^{\top}\to 0\tag{32}$$

does not imply that $(\tilde{x}_{\mathbf{i}}(t),\tilde{y}_{\mathbf{i}}(t))\to 0$ because of the rank deficiency of $[\cos(\theta_{\mathbf{i}}),\sin(\theta_{\mathbf{i}})]$, a persistently exciting $\theta_{\mathbf{i}}(t)$ will guarantee that the position error $(\tilde{x}_{\mathbf{i}}(t),\tilde{y}_{\mathbf{i}}(t))\to 0$ as $t\to\infty$.

Proposition 2. *Assume that the velocity error vector $\tilde{v}_{\mathbf{i}}(t)\in L_1\cap L_\infty$, and that $\omega_{\mathbf{i}}(t)$ is persistently exciting ($\omega_{\mathbf{i}}\in$ PE), that is, there exist constants $T_{\mathbf{i}},\mu_{\mathbf{i}}>0$ such that*

$$\int_t^{t+T_{\mathbf{i}}}\omega_{\mathbf{i}}(\tau)^2\mathrm{d}\tau\ge\mu_{\mathbf{i}},\quad\forall t\ge 0.\tag{33}$$

Then, $\tilde{x}_{\mathbf{i}}^{b}(t)\to 0$ as $t\to\infty$ implies that $(\tilde{x}_{\mathbf{i}}(t),\tilde{y}_{\mathbf{i}}(t))\to 0$ as $t\to\infty$.

Proof. Note that $J(q_{\mathbf{i}})$ is an orthogonal matrix, and $\tilde{v}_{\mathbf{i}}(t)\in L_1\cap L_\infty$ implies that $(\dot{q}_{\mathbf{i}}(t)-\dot{\bar{q}}_{\mathbf{i}}(t))\in L_1\cap L_\infty$. Also, $\tilde{v}_{\mathbf{i}}(t)\to 0$ implies that $(\dot{q}_{\mathbf{i}}(t)-\dot{\bar{q}}_{\mathbf{i}}(t))\to 0$ as $t\to\infty$. Thus, by integrating both sides, we conclude that $(\tilde{x}_{\mathbf{i}}(t),\tilde{y}_{\mathbf{i}}(t))\to$ const. Now, consider the following equation

$$c_1\cos(\theta_{\mathbf{i}})+c_2\sin(\theta_{\mathbf{i}})=0,\tag{34}$$

where c_1,c_2 are constants. If one of c_1 and c_2 is non-zero, then Eq. (34) has only isolate solutions $\theta_{\mathbf{i}}=$ const. On the other hand, by the filter property of persistently exciting signals, $\omega_{\mathbf{i}}\in$ PE implies that $\theta_{\mathbf{i}}$ does not converge to a constant as $t\to\infty$. Thus, by contradiction and the continuity of (32), we conclude that the position error $(\tilde{x}_{\mathbf{i}}(t),\tilde{y}_{\mathbf{i}}(t))\to 0$ as $t\to\infty$. □

It follows from Propositions 1 and 2 that if the angular velocity of the vehicle **i** is PE, then the modified PD+ controller (30) with $u_{\mathbf{i}}\equiv 0$ can be used to solve the formation tracking problem. However, in the cases of formation stabilization and formation tracking of a straight line, the angular velocity of the vehicle **i** converges to zero and thus the PE property is lost. In this case, we will use $u_{\mathbf{i}}$ as a "PE perturbation" on the angular motion to prevent $(\tilde{x}_{\mathbf{i}}(t),\tilde{y}_{\mathbf{i}}(t))$ converging to a non-zero constant. The new control input $u_{\mathbf{i}}$ is defined as

$$u_{\mathbf{i}}=\begin{bmatrix}0 & 0 & \alpha_{\mathbf{i}}(t,\tilde{y}_{\mathbf{i}}^{b})\end{bmatrix}^{\top},\tag{35}$$

where $\alpha_{\mathbf{i}}(t,\tilde{y}_{\mathbf{i}}^{b})=k_{\rho\mathbf{i}}\rho_{\mathbf{i}}(t)\tilde{y}_{\mathbf{i}}^{b}(t)$, $k_{\rho\mathbf{i}}>0$ is a constant, and the time-varying signal $\rho_{\mathbf{i}}(t)$ is PE, continuously differentiable, and bounded with bounded first derivative. Note that the excitation property of $\alpha_{\mathbf{i}}$ is reminiscent of $u\delta$-PE with respect to $\tilde{y}_{\mathbf{i}}^{b}$ [39], i.e., for each $\delta>0$ there exist $T,\mu>0$ such that

$$\left|\tilde{y}_{\mathbf{i}}^{b}(t)\right|>\delta\Rightarrow\int_t^{t+T}\alpha_{\mathbf{i}}(\tau,\tilde{y}_{\mathbf{i}}^{b})^2\mathrm{d}\tau>\mu,\quad\forall t\ge 0.\tag{36}$$

The illustration of the modified PD+ controller (30) with $u\delta$-PE "perturbation" (35) in closed loop is shown in Fig. 2. It is noted that $\theta_{\mathbf{i}}(t)$ and $\omega_{\mathbf{i}}(t)$ should be considered as time-varying signals in the linear motion error dynamics, and for any $(\theta_{\mathbf{i}},\omega_{\mathbf{i}})\in L_\infty$, the linear motion error dynamics are globally asymptotically stable with respect to $(\tilde{x}_{\mathbf{i}}^{b},\tilde{v}_{x\mathbf{i}},\tilde{v}_{y\mathbf{i}})$ uniformly in $\tilde{y}_{\mathbf{i}}^{b}(0)$.

Proposition 3. *Consider the planar underactuated vehicle (1a), (1b) satisfying Assumption 1. Then, under the modified PD+ control law (30) and (35), the origin for the $(\tilde{x}_{\mathbf{i}}^{b},\tilde{y}_{\mathbf{i}}^{b},\tilde{\theta}_{\mathbf{i}},\tilde{v}_{x\mathbf{i}},\tilde{v}_{y\mathbf{i}},\tilde{\omega}_{\mathbf{i}})$-dynamics is UGAS.*

Proof. It follows from Proposition 1 that if $\alpha_{\mathbf{i}}(t,\tilde{y}_{\mathbf{i}}^{b})\equiv 0$, the $(\tilde{x}_{\mathbf{i}}^{b},\tilde{\theta}_{\mathbf{i}},\tilde{v}_{x\mathbf{i}},\tilde{v}_{y\mathbf{i}},\tilde{\omega}_{\mathbf{i}})$-subsystem is UGAS to its origin. Furthermore, due to the damping term $K_{d\mathbf{i}}$ in the PD+ control law (30), the angular motion dynamics is input-to-state stable (ISS) by considering $\alpha_{\mathbf{i}}(t,\tilde{y}_{\mathbf{i}}^{b})$ as an input, as shown in Fig. 2. It also follows from the proof in Proposition 2 that $\tilde{y}_{\mathbf{i}}^{b}(t)$ converges to a constant as $t\to\infty$. Now, assume that $\tilde{y}_{\mathbf{i}}^{b}(t)$ converges to a non-zero

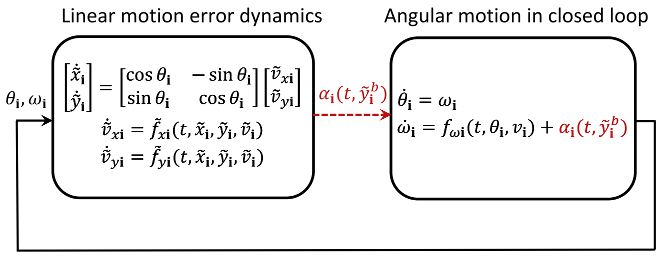


Fig. 2. Illustration of the modified PD+ controller (30) in closed loop with $u\delta$-PE "perturbation" (35).

constant. Then, (36) implies $\alpha_{\mathbf{i}} \in$ PE, and from the filter property we have $\omega_{\mathbf{i}}(t) \in$ PE. Then, it follows from Proposition 2 that $(\tilde{x}_{\mathbf{i}}(t), \tilde{y}_{\mathbf{i}}(t)) \to 0$ as $t \to \infty$, which contradicts the assumption that $\tilde{y}_{\mathbf{i}}^b(t)$ converges to a non-zero constant. Thus, we conclude that $\tilde{y}_{\mathbf{i}}^b(t) \to 0$ as $t \to \infty$ by contradiction. The UGAS of the origin comes from the ISS property when $\alpha_{\mathbf{i}}(t, \tilde{y}_{\mathbf{i}}^b) \to 0$ as $t \to \infty$, which completes the proof. □

Our main result comes from the previous rationale.

Theorem 3. *Consider a network of heterogeneous planar underactuated vehicles satisfying Assumptions 1 and 2. Then, the formation is achieved under the modified PD+ control law* (30) and (35) *if the directed communication graph $\mathcal{G}$ contains a spanning tree.*

Proof. By the assumption of the spanning tree topology in the communication graph and using Proposition 3, an immediate consequence of the claim is that for each vehicle $\mathbf{i}$ in the group, the origin for the $(\tilde{x}_{\mathbf{i}}^b, \tilde{y}_{\mathbf{i}}^b, \tilde{\theta}_{\mathbf{i}}, \tilde{v}_{x\mathbf{i}}, \tilde{v}_{y\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$-dynamics is UGAS. It follows from the converse Lyapunov theorem that there exist a continuously differentiable function $\mathbf{V}_{\mathbf{i}} : \mathbb{R} \times \mathbb{R}^6 \to \mathbb{R}_{\geq 0}$, $\varphi_{1\mathbf{i}}, \varphi_{2\mathbf{i}} \in \mathcal{K}_\infty$, and a positive definite function $\mathbf{W}_{\mathbf{i}}$ such that

$$\varphi_{1\mathbf{i}}\left(\left|(\tilde{q}_{\mathbf{i}}^b, \tilde{v}_{\mathbf{i}})\right|\right) \leq \mathbf{V}_{\mathbf{i}}\left(t, \tilde{q}_{\mathbf{i}}^b, \tilde{v}_{\mathbf{i}}\right) \leq \varphi_{2\mathbf{i}}\left(\left|(\tilde{q}_{\mathbf{i}}^b, \tilde{v}_{\mathbf{i}})\right|\right),$$
$$\dot{\mathbf{V}}_{\mathbf{i}} \leq -\mathbf{W}_{\mathbf{i}}\left((\tilde{q}_{\mathbf{i}}^b, \tilde{v}_{\mathbf{i}})\right).$$

Then, define the Lyapunov candidate

$$\mathbf{V} := \sum_{\mathbf{i}\in\mathcal{V}}\sum_{\mathbf{j}\in\mathcal{N}_{\mathbf{i}}} a_{\mathbf{ij}}\mathbf{V}_{\mathbf{i}}. \tag{37}$$

Note that if the communication graph contains a spanning tree, then the Lyapunov candidate $\mathbf{V}$ covers all the agents in the network. Taking the time derivative along the trajectories of the closed-loop system, we have that

$$\dot{\mathbf{V}} \leq -\sum_{\mathbf{i}\in\mathcal{V}}\sum_{\mathbf{j}\in\mathcal{N}_{\mathbf{i}}} a_{\mathbf{ij}}\mathbf{W}_{\mathbf{i}}\left((\tilde{q}_{\mathbf{i}}^b, \tilde{v}_{\mathbf{i}})\right). \tag{38}$$

Thus, the formation error converges to zero as $t \to \infty$, and we conclude that the formation is achieved if the communication graph contains a spanning tree. □

Remark 4. It is noted that, for each agent, the local motion information (e.g., orientation angle, linear velocity, and angle velocity, etc.) can be measured by its own onboard sensors (e.g., gyroscope, speedometer, IMU, etc.), and then sent to its followers via the communication network. The relative position information can be directly measured by the onboard sensors of its followers (e.g., Lidar, camera, etc.) in its own body-fixed frame, and then can be converted to the error in the global coordinates $(q_{\mathbf{i}} - q_{\mathbf{j}})$ by multiply the orthogonal transformation matrix $J(\theta_{\mathbf{i}})$ given in (2).

4. Applications and simulation results

In this section we present specific forms of the general EL model (1a), (1b) for various vehicles, and present numerical simulations to illustrate the effectiveness of the proposed formation control law. Vehicles chosen are underactuated surface vessels and ground mobile robots. The above combination of the vehicles can be used in robotic manipulators installed on boards of surface vessels and ground vehicles for coordinated load carrying in canals, for surveillance operations where coordination between the units on bodies of water (particularly rivers) and on the ground is needed, and for military operations to increase the striking force from multiple sources in the sea and on the ground, to name a few examples.

4.1. Applications

Underactuated Surface Vessels. The EL equations for an underactuated surface vessel model with nonlinear hydrodynamic damping are given by (1a), (1b) with

$$M = \begin{bmatrix} m_{11} & 0 & 0\\ 0 & m_{22} & 0\\ 0 & 0 & m_{33}\end{bmatrix}, C(v) = \begin{bmatrix} 0 & 0 & -m_{22}v_y\\ 0 & 0 & m_{11}v_x\\ m_{22}v_y & -m_{11}v_x & 0\end{bmatrix},$$
$$D(v) = \begin{bmatrix} d_{11}|v_x|^{\alpha_{11}} & 0 & 0\\ 0 & d_{22}|v_y|^{\alpha_{22}} & 0\\ 0 & 0 & d_{33}|\omega|^{\alpha_{33}}\end{bmatrix}, G = \begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix},$$

where $m_{ii} > 0$; $d_{ii} > 0$ and $0 \leq \alpha_{ii} < 1$ for $i = 1, 2, 3$ [3,14]. This model is also applicable to linear hydrodynamic damping with $\alpha_{ii} = 0$, which is the model used in [2,24–26]. Note that the conditions in Assumption 1 can be verified directly and are satisfied for this model.

Wheeled Mobile Robots. Due to the nonholonomic constraints, the dimensions of the tangent (velocity) space is reduced. The EL equations for a nonholonomic mobile robot model are given by (1a), (1b) with

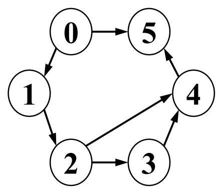
$$M = \begin{bmatrix}\tilde{m} & 0\\ 0 & \tilde{I}\end{bmatrix}, C(v) = \begin{bmatrix}0 & -md\omega\\ md\omega & 0\end{bmatrix}, D(v) = 0, G = \begin{bmatrix}\frac{1}{r} & 0\\ 0 & \frac{a}{r}\end{bmatrix},$$

where $\tilde{m} = m + 2J/r^2$, $\tilde{I} = I + md^2 + a^2J/r^2$, and $m, d, I, J, a, r > 0$ are constants [13,15,16]. Although there is no v_y-dynamics in the model and the damping matrix $D(v)$ is zero, the nonholonomic constraint $v_y = d\omega$ suggests that the damping term introduced by the control law $K_{d\mathbf{i}}$ makes the dynamic equations output strictly passive. Thus, the modified PD+ control law (30), (35) can be applied to this model directly, and the UGAS for the v_y-subsystem comes directly from the linear relationship between the ω-dynamics and the v_y-dynamics.

4.2. Numerical simulations

Let us consider a group of six planar underactuated vehicles with the indices $\mathbf{0} - \mathbf{5}$. Agent $\mathbf{0}$ is the leader and agents $\mathbf{1} - \mathbf{5}$ are the followers with the communication topology graph and the weighted adjacency matrix as shown in Fig. 3. Note that we assume that the communication from agent $\mathbf{0}$ to agent $\mathbf{5}$, and the communication from agent $\mathbf{2}$ to agent $\mathbf{4}$ are twice as important as the other communication links. We assume that agents $\mathbf{0}, \mathbf{1}$ are surface vessels with linear hydrodynamic damping whose parameters are given as

$$m_{11,\mathbf{i}} = 1.412,\quad m_{22,\mathbf{i}} = 1.982,\quad m_{33,\mathbf{i}} = 0.354,$$
$$d_{11,\mathbf{i}} = 3.436,\quad d_{22,\mathbf{i}} = 12.99,\quad d_{33,\mathbf{i}} = 0.864;$$



$$\mathcal{A}=\begin{bmatrix}0&0&0&0&0&0\\1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&2&1&0&0\\2&0&0&0&1&0\end{bmatrix}$$

Fig. 3. Directed communication topology and the weighted adjacency matrix used in the simulations.

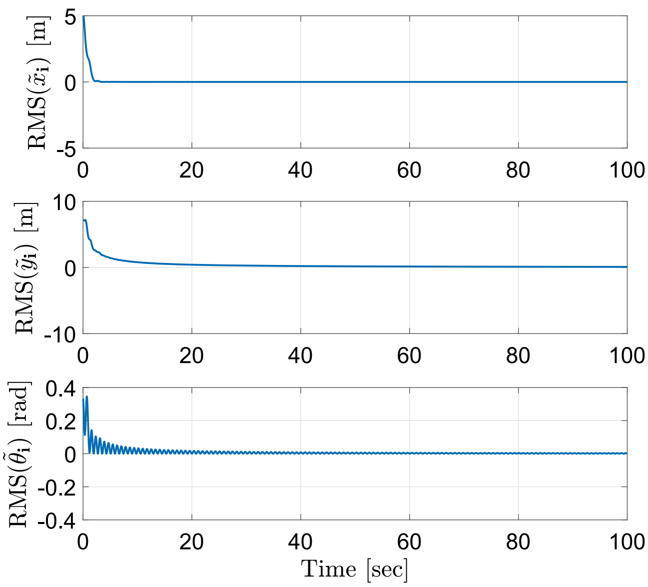


Fig. 4. Time history of the RMS errors of the formation stabilization.

agents **2**, **3** are surface vessels with nonlinear hydrodynamic damping whose parameters are given as

$$\begin{aligned}&m_{11,\mathbf{i}}=1.317, && m_{22,\mathbf{i}}=3.832, && m_{33,\mathbf{i}}=0.926,\\ &d_{11,\mathbf{i}}=5.252, && d_{22,\mathbf{i}}=14.14, && d_{33,\mathbf{i}}=2.262,\\ &\alpha_{11,\mathbf{i}}=0.510, && \alpha_{22,\mathbf{i}}=0.747, && \alpha_{33,\mathbf{i}}=1.592;\end{aligned}$$

agents **4**, **5** are nonholonomic mobile robots whose parameters are given as

$$\begin{aligned}&m_{\mathbf{i}}=3.0, && I_{\mathbf{i}}=0.025, && J_{\mathbf{i}}=6\times10^{-6},\\ &a_{\mathbf{i}}=0.33, && d_{\mathbf{i}}=0.08, && r_{\mathbf{i}}=0.05.\end{aligned}$$

All the parameters are given in SI units. The desired geometric pattern in formation is assumed to be a regular hexagon with the side length $h=2$, i.e., $(d^x_{\mathbf{10}},d^y_{\mathbf{10}})=(-1,-\sqrt{3})$, $(d^x_{\mathbf{21}},d^y_{\mathbf{21}})=(1,-\sqrt{3})$, $(d^x_{\mathbf{32}},d^y_{\mathbf{32}})=(0,2)$, $(d^x_{\mathbf{43}},d^y_{\mathbf{43}})=(1,\sqrt{3})$, $(d^x_{\mathbf{54}},d^y_{\mathbf{54}})=(-1,\sqrt{3})$. The vehicles are assumed to be initially stationary at the coordinates

$$\begin{aligned}&q_{\mathbf{0}}(0)=(0,0,0), && q_{\mathbf{1}}(0)=(-5,-5,0), && q_{\mathbf{2}}(0)=(-2,-6,1),\\ &q_{\mathbf{3}}(0)=(3,-5,1), && q_{\mathbf{4}}(0)=(5,-5,1), && q_{\mathbf{5}}(0)=(5,2,0).\end{aligned}$$

Formation Stabilization. In the first simulation, we assume that the desired configuration for the group leader **0** is at the origin for all times $t\geq 0$. The control parameters are selected as $K_{p\mathbf{i}}=\operatorname{diag}\{5,5,5\}$, $K_{d\mathbf{i}}=\operatorname{diag}\{4,4,4\}$, $k_{\rho\mathbf{i}}=2$ and $\rho_{\mathbf{i}}(t)=\sin(2t)$ for all $\mathbf{i}\in\mathcal{V}$.

The simulation results are shown in Figs. 4–5, where the root mean square (RMS) error shown in Fig. 4 is of the form $\mathrm{RMS}([\cdot]_{\mathbf{i}})=(\frac{1}{n}\sum_{i=1}^{n}[\cdot]^2_{\mathbf{i}})^{1/2}$. It can be seen from the figures that the formation errors approach zero after 40 s. As shown in Figs. 4–5, firstly, each vehicle converges to a small neighborhood of the desired formation position very fast. Then, it converges to the desired formation position with oscillation, and this convergence phase is slow. This oscillation is due to the $u\delta$-PE term $\alpha_{\mathbf{i}}$ introduced in the control law, and it is a common phenomenon in stabilization of nonholonomic and underactuated systems via smooth time-varying feedbacks.

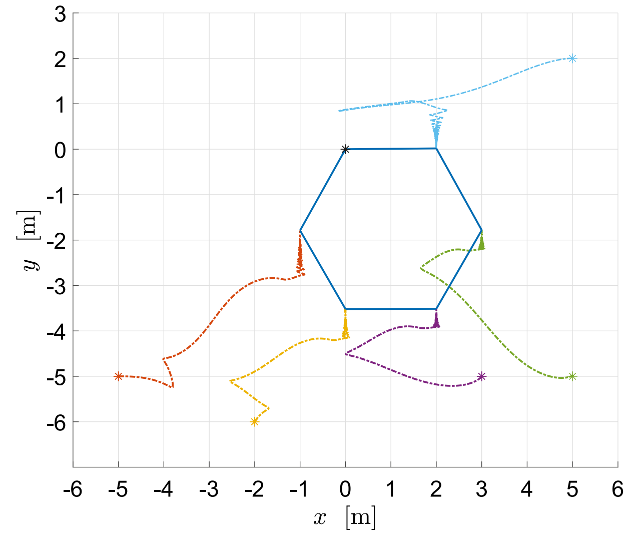


Fig. 5. Position paths in the $\{XY\}$ frame of the formation stabilization.

Formation Tracking. In the second simulation, we assume that the desired path for the group leader **0** is a U-shape function, i.e.,

$$(x_{\mathrm{d}}(t),y_{\mathrm{d}}(t))=\begin{cases}(0.5t,0), & 0\leq t<40,\\ (20+5\sin(\frac{\pi t}{20}),5-5\cos(\frac{\pi t}{20})), & 40\leq t<60,\\ (20-0.5(t-60),10), & 60\leq t.\end{cases}$$

The control parameters are selected as $K_{p\mathbf{i}}=\operatorname{diag}\{8,8,8\}$, $K_{d\mathbf{i}}=\operatorname{diag}\{4,4,4\}$, $k_{\rho\mathbf{i}}=4$ and $\rho_{\mathbf{i}}(t)=\sin(4t)$ for all $\mathbf{i}\in\mathcal{V}$.

The simulation results are shown in Figs. 6–7. It can be seen from the figures that all formation tracking errors approach zero with satisfactory performance. It is noted that the proposed control law is essentially a PD-type controller. Thus, it is reasonable to expect a better performance and robustness with high control gains. Based on the above simulations, the effectiveness of the proposed formation control scheme is verified.

5. Concluding remarks

In this work, we presented a distributed control framework to *simultaneously* address the formation stabilization and tracking control problem for heterogeneous planar underactuated vehicle networks without global position measurements. The vehicles in the network are modeled as generic EL systems and are allowed to have identical or non-identical dynamics. The control design is developed based on partial stability theory, Matrosov's theorem, and $u\delta$-PE, and guarantees UGAS for the origin of the closed-loop system. The proposed controller has a PD+ form and is relatively simple compared to existing controllers in the literature that, in addition, solve the stabilization and tracking problems *separately*. Thus, it is practical and easy to implement. Further research is being carried out to extend the proposed method to cooperative control of three-dimensional vehicles such as quadcopters. We also plan to extend this work to address dynamic communication topology and communication delays, which are significant and prevalent in multi-agent networks.

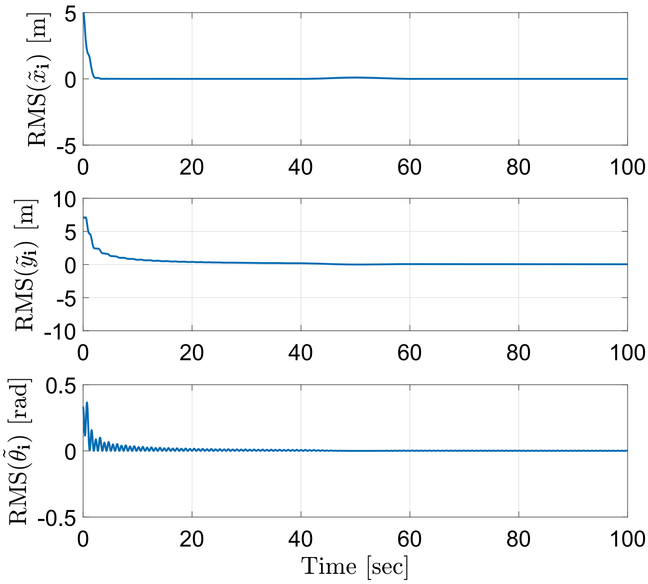


Fig. 6. Time history of the RMS errors of the formation tracking.

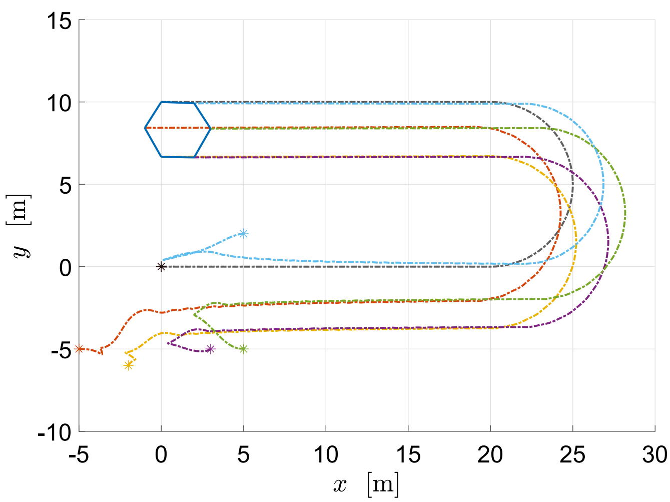


Fig. 7. Position paths in the $\{XY\}$ frame of the formation tracking.

CRediT authorship contribution statement

Bo Wang: Lead student author, Developed distributed control, Methodology, Worked out simulations. **Hashem Ashrafiuon:** Concept idea, Student supervision, Verification of the results. **Sergey Nersesov:** Concept idea, Student supervision, Verification of the results, Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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