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Formation Control for Heterogeneous Spatial Underactuated Vehicles Using Bearing Measurements ⋆

Bo Wang * Sergey Nersesov * Hashem Ashrafiuon *

* *Department of Mechanical Engineering, Villanova University, Villanova, PA 19085, USA (e-mail: {bwang6, sergey.nersesov, hashem.ashrafiuon}@villanova.edu)*

Abstract: In this work, we solve the distributed formation control problem for heterogeneous spatial underactuated vehicles subject to switching topologies. We consider the spatial rigid body model of vehicles with one translational actuator for propulsion and three rotational actuators. A finite-time sliding mode observer is designed to estimate the ranges between vehicles based on the bearing measurements. A generalized Slotine-Li transformation is proposed to define continuous reference velocity trajectories under switching topologies. Based on the cascade structure, a distributed formation protocol is presented which guarantees the global asymptotic convergence for the closed-loop system. Numerical simulations on a group of spatial underactuated vehicles including quadcopters and underwater vehicles are presented.

Keywords: Formation control, spatial underactuated vehicles, heterogeneous networks.

1. INTRODUCTION

The cooperative control problem for multiple vehicles has received great attention in the past two decades. The advantages of multi-vehicle systems over single vehicles include higher efficiency, robustness, and flexibility (Ren and Beard, 2008). Numerous cooperative control strategies have been proposed in the literature for autonomous vehicles modeled by single-integrator dynamics (Olfati-Saber and Murray, 2004; Ren and Beard, 2008), double-integrator dynamics (Ren, 2008), and fully-actuated Euler-Lagrangian (EL) systems (Mei et al., 2011).

Most of the vehicles in practice are *underactuated*. That is, the vehicle has fewer number of independent actuators than its degrees of freedom (DOF). The approaches developed for integrator dynamics and for fully-actuated EL systems cannot be directly applied to underactuated vehicle networks. Furthermore, a multi-vehicle system usually contains different types of vehicles. It is more practical if a group of vehicles can cooperate with each other regardless of the structures of their dynamic models. However, there has not been much effort to develop cooperative control approaches that can be applied to heterogeneous underactuated multi-agent systems in the literature. Moreover, an important theme in multi-agent control systems is *decentralization*, namely, *distributed algorithms*, where each agent senses the *relative* configuration variables of its neighbors with respect to its *local* coordinate system (Oh et al., 2015). For distributed controllers, cameras and inertial measurement units (IMUs) are usually the preferred onboard sensors compared to LiDARs due to lower weight and cost. These sensors can measure bearing angles, postures, velocities and accelerations. Hence, the ranges or relative positions need to be estimated. Another requirement for distributed cooperative control of multi-agent systems is communication (Ren and Beard, 2008). Switching communication topologies and communication delays are common in a communication network, and thus, they should be considered when designing a controller.

In Zhang et al. (2021), the formation–containment control problem was considered for heterogeneous underactuated autonomous underwater vehicles (AUVs) in three-dimensional space based on a simplified 5-DOF model. In Mu et al. (2017); Mu and Shi (2018), an integral sliding mode control law and a linear quadratic regulation (LQR) consensus protocol were proposed for heterogeneous multi-vehicle systems consisting of quadrotors and wheeled mobile robots based on the linearized models. Recently, in Wang and Ahn (2021), a coordinated trajectory tracking controller was proposed for the marine aerial-surface heterogeneous system composed by a quadrotor and a (fully-actuated) surface vehicle based on cascaded system theory and Lyapunov analysis. Nevertheless, in the above mentioned works, the vehicle models in the heterogeneous networks are either simplified, linearized, or partially assumed to be fully-actuated.

In this work, we solve the distributed formation control problem for a class of *heterogeneous* spatial underactuated vehicle networks with directed communication graphs. We consider generic spatial vehicle model with two degrees of underactuation, which includes underwater and aerial vehicles with one translational actuator and three rotational actuators. Based on the cascaded structure, the formation controller guarantees the global asymptotic convergence for the closed-loop system. We prove that switching topologies do not matter if the communication graphs contains a directed spanning tree. The proposed

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control protocol requires the *bearing angle information* of the neighbor vehicles, instead of requiring the relative positions. Using the distributed sliding mode observers, the ranges between the vehicle and its neighbors are estimated in finite time. The proposed distributed control law requires only neighbor-to-neighbor information exchange, and all the measurements are based on onboard sensors.

Notations. Let $\mathbb{R}_{\geqslant 0}$ be the set of all non-negative real numbers; $\mathbb{Z}_{\geqslant 0}$ the set of all non-negative integers; $I_n \in \mathbb{R}^{n\times n}$ the identity matrix; $\mathbb{S}^n$ the n-sphere, i.e., $\mathbb{S}^n = \{x \in \mathbb{R}^{n+1} : |x| = 1\}$; s the differential operator, i.e., $s = \frac{\mathrm{d}}{\mathrm{d}t}[\cdot]$. We use the abbreviation $s_{(\cdot)} = \sin(\cdot)$, $c_{(\cdot)} = \cos(\cdot)$, and $t_{(\cdot)} = \tan(\cdot)$. Given $a = [a_1, a_2, a_3]^\top \in \mathbb{R}^3$, we define the operator $(\cdot)_\times$ as

$$a_\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}.$$

2. PRELIMINARIES AND PROBLEM STATEMENT

2.1 Model of spatial underactuated vehicles

Consider a network of N heterogeneous spatial underactuated vehicles, where the agents are numbered $\mathbf{i} = 1, \ldots, N$ with $\mathbf{1}$ representing the group leader and $\mathbf{2}, \ldots, \mathbf{N}$ representing the followers. Each vehicle is modeled as a 6-DOF rigid body moving in three-dimensional Euclidean space. Let $\{\mathcal{I}\}$ denote an earth-fixed inertial frame, and $\{\mathcal{B}_\mathbf{i}\}$ the body-fixed frame attached to vehicle $\mathbf{i}$, where the origin is located at the center of mass of the vehicle, as shown in Fig. 1. The position of vehicle $\mathbf{i}$ in the earth-fixed frame $\{\mathcal{I}\}$ is represented by $\xi_\mathbf{i} = [x_\mathbf{i}, y_\mathbf{i}, z_\mathbf{i}]^\top$, and the attitude is represented by the Euler angles $\eta_\mathbf{i} = [\phi_\mathbf{i}, \theta_\mathbf{i}, \psi_\mathbf{i}]^\top$ of $\{\mathcal{B}_\mathbf{i}\}$ relative to $\{\mathcal{I}\}$, where $\phi_\mathbf{i}, \theta_\mathbf{i}, \psi_\mathbf{i}$ represent the roll, pitch, and yaw angles, respectively. Let $v_\mathbf{i} = [v_{x\mathbf{i}}, v_{y\mathbf{i}}, v_{z\mathbf{i}}]^\top$ and $\omega_\mathbf{i} = [\omega_{x\mathbf{i}}, \omega_{y\mathbf{i}}, \omega_{z\mathbf{i}}]^\top$ denote the linear and angular velocities of vehicle $\mathbf{i}$ in its body-fixed frame, respectively. The kinematics of vehicle $\mathbf{i}$ is described by

$$\begin{bmatrix} \dot{\xi}_\mathbf{i} \\ \dot{\eta}_\mathbf{i} \end{bmatrix} = \begin{bmatrix} R(\eta_\mathbf{i}) & 0 \\ 0 & T(\eta_\mathbf{i}) \end{bmatrix} \begin{bmatrix} v_\mathbf{i} \\ \omega_\mathbf{i} \end{bmatrix} \tag{1}$$

where $R(\cdot)$ is the rotation matrix given by

$$R(\eta) = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix},$$

and the matrix $T(\cdot)$ is given by

$$T(\eta) = \begin{bmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & \dfrac{s_\phi}{c_\theta} & \dfrac{c_\phi}{c_\theta} \end{bmatrix}.$$

Note that the matrix $T(\eta)$ becomes singular when $\theta = \pm\pi/2$, and thus, we restrict the use of Euler angles to $|\phi_\mathbf{i}| < \pi/2$ and $|\theta_\mathbf{i}| < \pi/2$ to avoid aggressive maneuvers and singularity (Fetzer et al., 2021).

We consider the spatial vehicle model with two degrees of underactuation. More precisely, we assume that each vehicle has only one control thrust (force) and three control torques. The dynamic EL model of vehicle $\mathbf{i}$ can be written as

$$\begin{aligned} m_\mathbf{i}\dot{v}_\mathbf{i} + \omega_\mathbf{i} \times (m_\mathbf{i} v_\mathbf{i}) + D_{v\mathbf{i}} v_\mathbf{i} &= F_\mathbf{i} + R(\eta_\mathbf{i})^\top G_\mathbf{i}, \\ I_\mathbf{i}\dot{\omega}_\mathbf{i} + \omega_\mathbf{i} \times (I_\mathbf{i}\omega_\mathbf{i}) + D_{\omega\mathbf{i}}\omega_\mathbf{i} &= \tau_\mathbf{i}, \end{aligned} \tag{2}$$

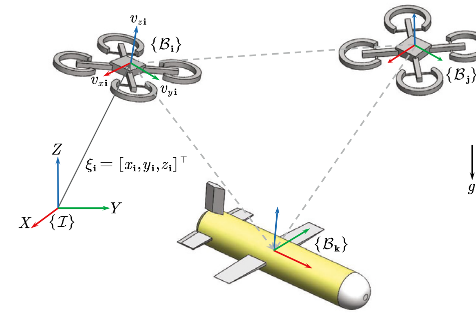


Fig. 1. Illustration of the leader-follower formation of heterogeneous spatial underactuated vehicle networks.

where $m_\mathbf{i}$ is the total mass of the vehicle; $I_\mathbf{i} \in \mathbb{R}^{3\times 3}$ is the diagonal inertia matrix; $D_{v\mathbf{i}}, D_{\omega\mathbf{i}} \in \mathbb{R}^{3\times 3}$ are constant, positive semi-definite damping matrices; $F_\mathbf{i}$ is the control thrust force; $G_\mathbf{i} = [0, 0, G_{z\mathbf{i}}]^\top$ is the total force of gravity and the buoyancy (if exists); $\tau_\mathbf{i} = [\tau_{\phi\mathbf{i}}, \tau_{\theta\mathbf{i}}, \tau_{\psi\mathbf{i}}]^\top$ is the control torque. Due to the underactuation, the vehicle model only has one control thrust, and without any loss of generality, we assume that the control thrust is in the direction of one of the three body-fixed axes, i.e., $F_\mathbf{i} = [F_{x\mathbf{i}}, 0, 0]^\top$, $F_\mathbf{i} = [0, F_{y\mathbf{i}}, 0]^\top$, or $F_\mathbf{i} = [0, 0, F_{z\mathbf{i}}]^\top$. It should be noted that the full nonlinear vehicle model (1)-(2) can represent a wide class of spatial underactuated vehicles including AUVs ($F_\mathbf{i} = [F_{x\mathbf{i}}, 0, 0]^\top$) and quadrotors ($F_\mathbf{i} = [0, 0, F_{z\mathbf{i}}]^\top$) (Fetzer et al., 2021).

Taking time derivative of (1), substituting (2), and using the properties that $R(\eta_\mathbf{i})^\top = R(\eta_\mathbf{i})^{-1}$, $\dot{R}(\eta_\mathbf{i}) = R(\eta_\mathbf{i})(\omega_\mathbf{i})_\times$, and $(\omega_\mathbf{i})_\times v_\mathbf{i} = \omega_\mathbf{i} \times v_\mathbf{i}$, we obtain the equations of motion in the earth-fixed frame:

$$\ddot{\xi}_\mathbf{i} = R(\eta_\mathbf{i})u_\mathbf{i} + \frac{G_\mathbf{i}}{m_\mathbf{i}} - D_{\xi\mathbf{i}}(\eta_\mathbf{i})\dot{\xi}_\mathbf{i}, \tag{3}$$

$$\ddot{\eta}_\mathbf{i} = \tilde{\tau}_\mathbf{i}, \tag{4}$$

where $D_{\xi\mathbf{i}}(\eta_\mathbf{i}) = (1/m_\mathbf{i})R(\eta_\mathbf{i})D_{v\mathbf{i}}R(\eta_\mathbf{i})^\top$; $u_\mathbf{i} = F_\mathbf{i}/m_\mathbf{i}$ and $\tilde{\tau}_\mathbf{i} = \dot{T}(\eta_\mathbf{i})\omega_\mathbf{i} - T(\eta_\mathbf{i})I_\mathbf{i}^{-1}[\omega_\mathbf{i} \times (I_\mathbf{i}\omega_\mathbf{i}) + D_{\omega\mathbf{i}}\omega_\mathbf{i} - \tau_\mathbf{i}]$ are the new control inputs. Note that $u_\mathbf{i} = [u_{x\mathbf{i}}, 0, 0]^\top, [0, u_{y\mathbf{i}}, 0]^\top$, or $[0, 0, u_{z\mathbf{i}}]^\top$ according to the specific configuration of the thrust actuator, where $u_{(\cdot)\mathbf{i}} = F_{(\cdot)\mathbf{i}}/m_\mathbf{i}$.

2.2 Notions from graph theory

The information exchange among the N vehicles is modeled as a directed graph $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t), \mathcal{A}(t))$, where $\mathcal{V} = \{\mathbf{1}, \ldots, \mathbf{N}\}$ is the vertex set; $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set; and $\mathcal{A}(t) \in \mathbb{R}^{N\times N}$ is the weighted adjacency matrix. The set of neighboring nodes is denoted by $\mathcal{N}_\mathbf{i}(t) = \{\mathbf{j} \in \mathcal{V} : (\mathbf{i}, \mathbf{j}) \in \mathcal{E}_t\}$, where $(\mathbf{i}, \mathbf{j})$ represents that node $\mathbf{i}$ obtains information from node $\mathbf{j}$ via communication. The weighted adjacency matrix $\mathcal{A}(t) = [a_{\mathbf{ij}}(t)]$ is defined as $a_{\mathbf{ij}}(t) > 0$ if $\mathbf{j} \in \mathcal{N}_\mathbf{i}(t)$ and $a_{\mathbf{ij}}(t) = 0$ otherwise. We assume that the graph $\mathcal{G}(t)$ has no self-loop or loop for each $t \geqslant 0$.

Assumption 1. *(i) $\mathcal{A}(t)$ is is piecewise continuous for all $t \geqslant 0$; (ii) each nonzero entry $a_{\mathbf{ij}}(t)$ is bounded, i.e., there exist positive constants $\underline{a}, \bar{a}$ such that $\underline{a} < a_{\mathbf{ij}}(t) < \bar{a}$; (iii) Let $t_0 = 0$ and let $t_1, t_2, \ldots$ be the switching times for $\mathcal{A}(t)$. The directed switching graph $\mathcal{G}(t)$ has a directed spanning tree across each interval $[t_i, t_{i+1})$, $\forall i \in \mathbb{Z}_{\geqslant 0}$.*

2.3 Problem statement

The objective of formation control is to design a distributed controller for each follower agent such that it coordinates its motion relative to its neighbors, and the network asymptotically converges to a predefined geometric pattern. The desired geometric pattern of the vehicle network in terms of spatial positions is defined by a set of constant position offset vectors $\{d_{\mathbf{ij}} = [d^x_{\mathbf{ij}}, d^y_{\mathbf{ij}}, d^z_{\mathbf{ij}}]^\top \in \mathbb{R}^3 : \mathbf{i},\mathbf{j} \in \mathcal{V}, \mathbf{i} \neq \mathbf{j}\}$. To be more specific, under Assumption 1, we will design a controller for each follower (3)-(4) without global position measurements or relative range measurements such that: (i) the state trajectories of the closed-loop system are bounded for all $t \geqslant 0$; (ii) all the vehicles in the network can maintain a prescribed formation in the sense that for all $\mathbf{i} \in \mathcal{V}$,

$$\lim_{t\to+\infty} \sum_{\mathbf{j}\in\mathcal{N}_\mathbf{i}(t)} |\xi_\mathbf{i}(t) - \xi_\mathbf{j}(t) - d_{\mathbf{ij}}| = 0. \tag{5}$$

Lemma 1 (Ren and Beard (2008)). *Consider the single-integrator dynamics $\dot{x}_\mathbf{i} = u_\mathbf{i}$, where $x_\mathbf{i} \in \mathbb{R}^n$, $\mathbf{i} = 1, \ldots, N$ with the network communication graph satisfying Assumption 1. Then, under the control law*

$$u_\mathbf{i} = \frac{1}{\Xi_\mathbf{i}(t)} \sum_{\mathbf{j}\in\mathcal{N}_\mathbf{i}(t)} a_{\mathbf{ij}}(t)\left[\dot{x}_\mathbf{j} - \alpha(x_\mathbf{i} - x_\mathbf{j})\right], \tag{6}$$

where $\Xi_\mathbf{i}(t) = \sum_{\mathbf{j}\in\mathcal{N}_\mathbf{i}(t)} a_{\mathbf{ij}}(t)$, and $\alpha > 0$ is a constant, the consensus tracking problem is solved.

Lemma 2 (Sastry and Bodson (1989)). *For continuous differentiable signals $x, y : \mathbb{R}_{\geqslant 0} \to \mathbb{R}$, the following holds for any $\alpha > 0$*

$$\frac{\alpha}{s+\alpha}[xy] = y\frac{\alpha}{s+\alpha}[x] - \frac{1}{s+\alpha}\left[\dot{y}\frac{\alpha}{s+\alpha}[x]\right].$$

3. RANGE OBSERVER DESIGN

In this section, we present a distributed finite-time sliding mode observer for range estimation among spatial vehicles.

Consider a pair of agents $(\mathbf{i}, \mathbf{j})$, where agent $\mathbf{j}$ is the leader and agent $\mathbf{i}$ is the follower. In the body-fixed frame $\{\mathcal{B}_\mathbf{i}\}$, the relative position vector of agent $\mathbf{j}$ is denoted by

$$\zeta_{\mathbf{ij}} = R(\eta_\mathbf{i})^\top(\xi_\mathbf{j} - \xi_\mathbf{i}). \tag{7}$$

We assume that the measurable signal is the bearing angle of vehicle $\mathbf{j}$ in the body-fixed frame $\{\mathcal{B}_\mathbf{i}\}$. That is, we measure the projection of $\zeta_{\mathbf{ij}}$ on the unit sphere centered at the origin of $\{\mathcal{B}_\mathbf{i}\}$, i.e.,

$$\sigma_{\mathbf{ij}} = \frac{\zeta_{\mathbf{ij}}}{|\zeta_{\mathbf{ij}}|} \in \mathbb{S}^2. \tag{8}$$

The bearing angle $\sigma_{\mathbf{ij}}$ is well defined for all $|\zeta_{\mathbf{ij}}| \neq 0$. The problem is to estimate the range $r_{\mathbf{ij}} = |\zeta_{\mathbf{ij}}|$ based on the bearing angle $\sigma_{\mathbf{ij}}$, the attitude, and the velocity measurements.

To start with, we write the error dynamics in the body-fixed frame $\{\mathcal{B}_\mathbf{i}\}$. Note that

$$\dot{\overbrace{|\zeta_{\mathbf{ij}}|^2}} = 2|\zeta_{\mathbf{ij}}|\dot{\overbrace{|\zeta_{\mathbf{ij}}|}} = \dot{\overbrace{\zeta_{\mathbf{ij}}^\top\zeta_{\mathbf{ij}}}} = 2\zeta_{\mathbf{ij}}^\top\dot{\zeta}_{\mathbf{ij}} = 2r_{\mathbf{ij}}\dot{r}_{\mathbf{ij}},$$

and $\dot{r}_{\mathbf{ij}} = \sigma_{\mathbf{ij}}^\top\dot{\zeta}_{\mathbf{ij}}$. Taking time derivative of (7), and substituting (1), we obtain

$$\dot{r}_{\mathbf{ij}} = \sigma_{\mathbf{ij}}^\top\left[R(\eta_\mathbf{i})^\top R(\eta_\mathbf{j})v_\mathbf{j} - v_\mathbf{i}\right] = \sigma_{\mathbf{ij}}^\top w_{\mathbf{ij}}, \tag{9}$$

where $w_{\mathbf{ij}} = R(\eta_\mathbf{i})^\top R(\eta_\mathbf{j})v_\mathbf{j} - v_\mathbf{i}$, and we used the fact that the matrix $(\omega_\mathbf{i})_\times$ is skew-symmetric. Taking time derivative of (8), we have

$$\dot{\sigma}_{\mathbf{ij}} = -(\omega_\mathbf{i})_\times\sigma_{\mathbf{ij}} + \frac{1}{r_{\mathbf{ij}}}\left(I_3 - \sigma_{\mathbf{ij}}\sigma_{\mathbf{ij}}^\top\right)w_{\mathbf{ij}}. \tag{10}$$

Multiply $r_{\mathbf{ij}}$ and apply the stable filter $\alpha/(s+\alpha)$ with $\alpha > 0$ to both sides of (10) yields

$$\frac{\alpha}{s+\alpha}[r_{\mathbf{ij}}\dot{\sigma}_{\mathbf{ij}}] = \frac{\alpha}{s+\alpha}\left[-r_{\mathbf{ij}}(\omega_\mathbf{i})_\times\sigma_{\mathbf{ij}} + \left(I_3 - \sigma_{\mathbf{ij}}\sigma_{\mathbf{ij}}^\top\right)w_{\mathbf{ij}}\right]. \tag{11}$$

Applying Lemma 2, the left-hand side of (11) becomes

$$\alpha G_2[r_{\mathbf{ij}}\dot{\sigma}_{\mathbf{ij}}] = r_{\mathbf{ij}}G_1[\sigma_{\mathbf{ij}}] - G_2\left[\sigma_{\mathbf{ij}}^\top w_{\mathbf{ij}}G_1[\sigma_{\mathbf{ij}}]\right]. \tag{12}$$

where $G_1(s) = \alpha s/(s+\alpha)$ and $G_2(s) = 1/(s+\alpha)$. Substituting (12) into (11) and applying Lemma 2 again, we obtain

$$r_{\mathbf{ij}}\Phi_{\mathbf{ij}} = G_2\left[\sigma_{\mathbf{ij}}^\top w_{\mathbf{ij}}\Phi_{\mathbf{ij}}\right] + \alpha G_2\left[(I_3 - \sigma_{\mathbf{ij}}\sigma_{\mathbf{ij}}^\top)w_{\mathbf{ij}}\right], \tag{13}$$

where $\Phi_{\mathbf{ij}} = G_1[\sigma_{\mathbf{ij}}] + \alpha G_2\left[(\omega_\mathbf{i})_\times\sigma_{\mathbf{ij}}\right]$ is a continuous measurable signal.

Proposition 1. *Consider the dynamics (9)-(10) with input $w_{\mathbf{ij}}$. The sliding mode observer*

$$\begin{aligned}\dot{\hat{r}}_{\mathbf{ij}} = \sigma_{\mathbf{ij}}^\top w_{\mathbf{ij}} - \gamma\,\mathrm{sign}\big\{\Phi_{\mathbf{ij}}^\top\big(\Phi_{\mathbf{ij}}\hat{r}_{\mathbf{ij}} - G_2\left[\sigma_{\mathbf{ij}}^\top w_{\mathbf{ij}}\Phi_{\mathbf{ij}}\right]\\ -\alpha G_2\left[(I_3 - \sigma_{\mathbf{ij}}\sigma_{\mathbf{ij}}^\top)w_{\mathbf{ij}}\right]\big)\big\}\end{aligned} \tag{14}$$

$$\hat{\zeta}_{\mathbf{ij}} = \sigma_{\mathbf{ij}}\hat{r}_{\mathbf{ij}} \tag{15}$$

with $\gamma > 0$ provides a globally finite-time convergent estimate to the relative position error $\zeta_{\mathbf{ij}}$, i.e., there exists $T_r > 0$ such that $\hat{\zeta}_{\mathbf{ij}}(t) = \zeta_{\mathbf{ij}}(t)$ for all $t \geqslant T_r$, if the signal $\Phi_{\mathbf{ij}}^\top$ is persistently exciting (PE), i.e., there exist $\mu, T > 0$ such that

$$\int_t^{t+T}\Phi_{\mathbf{ij}}(s)^\top\Phi_{\mathbf{ij}}(s) \geqslant \mu, \quad \forall t \geqslant 0. \tag{16}$$

Proof. Define the estimation error $\tilde{r}_{\mathbf{ij}} = \hat{r}_{\mathbf{ij}} - r_{\mathbf{ij}}$. Substituting (13) into (14), the observation error dynamics are given by

$$\dot{\tilde{r}}_{\mathbf{ij}} = -\gamma\,\mathrm{sign}\left(\Phi_{\mathbf{ij}}^\top\Phi_{\mathbf{ij}}\right)\mathrm{sign}\left(\tilde{r}_{\mathbf{ij}}\right). \tag{17}$$

Consider the Lyapunov candidate $V(\tilde{r}_{\mathbf{ij}}) = |\tilde{r}_{\mathbf{ij}}|$, where its derivative is calculated as

$$\dot{V} = \begin{cases} -\gamma\,\mathrm{sign}\left(\Phi_{\mathbf{ij}}^\top\Phi_{\mathbf{ij}}\right), & \tilde{r}_{\mathbf{ij}} \neq 0 \\ 0, & \tilde{r}_{\mathbf{ij}} = 0. \end{cases} \tag{18}$$

For each $\tilde{r}_{\mathbf{ij}}(0) \neq 0$, we have, along trajectories, $\dot{V}(\tilde{r}_{\mathbf{ij}}(t)) = -\gamma$, if $\Phi_{\mathbf{ij}}(t)^\top\Phi_{\mathbf{ij}}(t) > 0$, and $\dot{V}(\tilde{r}_{\mathbf{ij}}(t)) = 0$, if $\Phi_{\mathbf{ij}}(t)^\top\Phi_{\mathbf{ij}}(t) = 0$. Due to the PE condition (16) and the continuity of $\Phi_{\mathbf{ij}}(t)$, for each time interval $[t, t+T]$, the measure of the set $\{s \in [t, t+T] : \Phi_{\mathbf{ij}}(s)^\top\Phi_{\mathbf{ij}}(s) \geqslant \mu/T\}$ must be (strictly) larger than zero. Define $l_{[a,b]}$ as the measure of the set $\{s \in [a, b] : \Phi_{\mathbf{ij}}(s)^\top\Phi_{\mathbf{ij}}(s) > 0\}$. We have, for all $t \geqslant 0$,

$$\begin{aligned} l_{[t,t+T]} &= \mathrm{meas}\{s \in [t, t+T] : \Phi_{\mathbf{ij}}(s)^\top\Phi_{\mathbf{ij}}(s) > 0\} \\ &> \mathrm{meas}\{s \in [t, t+T] : \Phi_{\mathbf{ij}}(s)^\top\Phi_{\mathbf{ij}}(s) \geqslant \mu/T\} > 0. \end{aligned}$$

Integrating both sides of (18) along trajectories yields $V(\tilde{r}_{\mathbf{ij}}(t)) = V(\tilde{r}_{\mathbf{ij}}(0)) - \gamma l_{[0,t]}t$. Therefore, for each $\tilde{r}_{\mathbf{ij}}(0) \neq 0$, there exists $T_r = V(\tilde{r}_{\mathbf{ij}}(0))/(\gamma l_{[0,t]})$ such that $V(\tilde{r}_{\mathbf{ij}}(T_r)) = 0$, which proves the global and finite-time convergence. □

4. FORMATION CONTROL DEVELOPMENT

Considering the formation objective (5), in addition to control the three position variables, one attitude variable also can be independently controlled. The other two attitude variables must be determined from the constraints imposed due to underactuation. The vehicle model (3)-(4) has three possible structural heterogeneities, which correspond to the three possible configurations of the thrust actuator. Introducing a virtual input $\nu_{\mathbf{i}} = [\nu_{x\mathbf{i}}, \nu_{y\mathbf{i}}, \nu_{z\mathbf{i}}]^\top \in \mathbb{R}^3$, we have

$$\ddot{\xi}_{\mathbf{i}} = \nu_{\mathbf{i}} + g_{\mathbf{i}}(\eta_{\mathbf{i}}, u_{\mathbf{i}}, \dot{\xi}_{\mathbf{i}}, \nu_{\mathbf{i}}), \tag{19}$$

where $g_{\mathbf{i}}(\eta_{\mathbf{i}}, u_{\mathbf{i}}, \dot{\xi}_{\mathbf{i}}, \nu_{\mathbf{i}}) = R(\eta_{\mathbf{i}})u_{\mathbf{i}} + G_{\mathbf{i}}/m_{\mathbf{i}} - D_{\xi\mathbf{i}}(\eta_{\mathbf{i}})\dot{\xi}_{\mathbf{i}} - \nu_{\mathbf{i}}$. The desired attitude signal $\eta_{\mathbf{i}d}(t) = [\phi_{\mathbf{i}d}(t), \theta_{\mathbf{i}d}(t), \psi_{\mathbf{i}d}(t)]^\top$ and $u_{\mathbf{i}}(t)$ are selected such that $g_{\mathbf{i}}(\eta_{\mathbf{i}d}(t), u_{\mathbf{i}}(t), \dot{\xi}_{\mathbf{i}}, \nu_{\mathbf{i}}) = 0$, for all $t \geqslant 0$. Specifically, denoting $\mu_{\mathbf{i}} = [\mu_{x\mathbf{i}}, \mu_{y\mathbf{i}}, \mu_{z\mathbf{i}}]^\top = D_{\xi\mathbf{i}}(\eta_{\mathbf{i}})\dot{\xi}_{\mathbf{i}} + \nu_{\mathbf{i}} - G_{\mathbf{i}}/m_{\mathbf{i}}$, the desired trajectories $\eta_{\mathbf{i}d}(t)$ and the thrust $u_{\mathbf{i}}(t)$ are selected such that $R(\eta_{\mathbf{i}d}(t))u_{\mathbf{i}}(t) = \mu_{\mathbf{i}}(t)$. For the three cases, we propose the attitude resolution as follows:

Case 1. ($u_{\mathbf{i}} = [u_{x\mathbf{i}}, 0, 0]^\top$; $\phi_{\mathbf{i}}$ is independently controlled.) Given $\phi_{\mathbf{i}d}(t) = \phi_{\mathbf{i}}(t)$ and $\nu_{\mathbf{i}}(t)$, the thrust and desired attitude signals are selected as

$$u_{x\mathbf{i}} = \sqrt{\mu_{x\mathbf{i}}^2 + \mu_{y\mathbf{i}}^2 + \mu_{z\mathbf{i}}^2}, \tag{20}$$

$$\theta_{\mathbf{i}d} = \arcsin\left(-u_{x\mathbf{i}}^{-1}\mu_{z\mathbf{i}}\right), \tag{21}$$

$$\psi_{\mathbf{i}d} = \arctan\left(\mu_{x\mathbf{i}}^{-1}\mu_{y\mathbf{i}}\right). \tag{22}$$

Case 2. ($u_{\mathbf{i}} = [0, u_{y\mathbf{i}}, 0]^\top$; $\theta_{\mathbf{i}}$ is independently controlled.) Given $\theta_{\mathbf{i}d}(t) = \theta_{\mathbf{i}}(t)$ and $\nu_{\mathbf{i}}(t)$, the thrust and desired attitude signals are selected as

$$u_{y\mathbf{i}} = \sqrt{\mu_{x\mathbf{i}}^2 + \mu_{y\mathbf{i}}^2 + \mu_{z\mathbf{i}}^2}, \tag{23}$$

$$\phi_{\mathbf{i}d} = \arcsin\left[\mu_{z\mathbf{i}}u_{y\mathbf{i}}^{-1}\sec(\theta_{\mathbf{i}d})\right], \tag{24}$$

$$\psi_{\mathbf{i}d} = \arccos\left[u_{y\mathbf{i}}\left(\mu_{x\mathbf{i}}\sin(\phi_{\mathbf{i}d})\sin(\theta_{\mathbf{i}d}) + \mu_{y\mathbf{i}}\cos(\phi_{\mathbf{i}d})\right)(\mu_{x\mathbf{i}}^2 + \mu_{y\mathbf{i}}^2)^{-1}\right]. \tag{25}$$

Case 3. ($u_{\mathbf{i}} = [0, 0, u_{z\mathbf{i}}]^\top$; $\psi_{\mathbf{i}}$ is independently controlled.) Given $\psi_{\mathbf{i}d}(t) = \psi_{\mathbf{i}}(t)$ and $\nu_{\mathbf{i}}(t)$, the thrust and desired attitude signals are selected as

$$u_{z\mathbf{i}} = \sqrt{\mu_{x\mathbf{i}}^2 + \mu_{y\mathbf{i}}^2 + \mu_{z\mathbf{i}}^2}, \tag{26}$$

$$\phi_{\mathbf{i}d} = \arcsin\left[u_{z\mathbf{i}}^{-1}\left(\mu_{x\mathbf{i}}\sin(\psi_{\mathbf{i}d}) - \mu_{y\mathbf{i}}\cos(\psi_{\mathbf{i}d})\right)\right], \tag{27}$$

$$\theta_{\mathbf{i}d} = \arctan\left[\mu_{z\mathbf{i}}^{-1}\left(\mu_{x\mathbf{i}}\cos(\psi_{\mathbf{i}d}) + \mu_{y\mathbf{i}}\sin(\psi_{\mathbf{i}d})\right)\right]. \tag{28}$$

In Slotine and Li (1987), an adaptive scheme was proposed for trajectory tracking control of fully-actuated Lagrangian systems with unknown parameters. The main idea of the Slotine-Li controller is to introduce a virtual "reference velocity", and then, a PD feedback is employed to steer the velocity variable to the "reference velocity". The Slotine-Li controller can be used to solve the consensus tracking problem for multi-agent systems. Consider N double-integrator systems, i.e., $\ddot{x}_{\mathbf{i}} = u_{\mathbf{i}}$ with $x_{\mathbf{i}} \in \mathbb{R}^n$, $\mathbf{i} = 1, \ldots, N$. Define the reference velocity $z_{\mathbf{i}}$, the sliding variable $s_{\mathbf{i}}$, and the control input $u_{\mathbf{i}}$ as

$$\begin{aligned} z_{\mathbf{i}} &= \frac{1}{\sum_{\mathbf{j}\in\mathcal{N}_{\mathbf{i}}} a_{\mathbf{ij}}} \sum_{\mathbf{j}\in\mathcal{N}_{\mathbf{i}}} a_{\mathbf{ij}}\left[\dot{x}_{\mathbf{j}} - (x_{\mathbf{i}} - x_{\mathbf{j}})\right], \\ s_{\mathbf{i}} &= \dot{x}_{\mathbf{i}} - z_{\mathbf{i}}, \\ u_{\mathbf{i}} &= \dot{z}_{\mathbf{i}} - k_{\mathbf{i}}s_{\mathbf{i}}, \end{aligned} \tag{29}$$

where $k_{\mathbf{i}} > 0$ is a constant control gain. The closed-loop dynamics on the sliding manifold $\{s_{\mathbf{i}} \equiv 0\}$ are given by

$$\dot{x}_{\mathbf{i}} = \frac{1}{\sum_{\mathbf{j}\in\mathcal{N}_{\mathbf{i}}} a_{\mathbf{ij}}} \sum_{\mathbf{j}\in\mathcal{N}_{\mathbf{i}}} a_{\mathbf{ij}}\left[\dot{x}_{\mathbf{j}} - (x_{\mathbf{i}} - x_{\mathbf{j}})\right]. \tag{30}$$

It follows from Lemma 1 that the consensus tracking problem is solved if the communication topology contains a directed spanning tree.

The algorithm (29) has a fatal flaw when the topology is dynamically changing. For instance, under switching topologies, $a_{\mathbf{ij}}(t)$ and the reference velocity $z_{\mathbf{i}}(t)$ are no longer continuous. Thus, the control law $u_{\mathbf{i}} = \dot{z}_{\mathbf{i}} - k_{\mathbf{i}}s_{\mathbf{i}}$ cannot be implemented because it involves the time derivative of a discontinuous term. To solve this problem, instead of defining the reference velocity $z_{\mathbf{i}}(t)$ as in (29), we define $z_{\mathbf{i}}(t)$ by integration. Consider the following algorithm

$$\dot{z}_{\mathbf{i}} = \frac{1}{\Xi_{\mathbf{i}}(t)} \sum_{\mathbf{j}\in\mathcal{N}_{\mathbf{i}}(t)} a_{\mathbf{ij}}(t)\left[\ddot{x}_{\mathbf{j}} - (\alpha+1)(\dot{x}_{\mathbf{i}} - \dot{x}_{\mathbf{j}}) - \alpha(x_{\mathbf{i}} - x_{\mathbf{j}})\right] \tag{31}$$

with $s_{\mathbf{i}}$ and $u_{\mathbf{i}}$ defined in (29), and $\alpha > 0$. It should be pointed that the reference velocity $z_{\mathbf{i}}(t)$ is differentiable due to the integration action, and thus, the control law $u_{\mathbf{i}} = \dot{z}_{\mathbf{i}} - k_{\mathbf{i}}s_{\mathbf{i}}$ is well defined. Consider the closed-loop dynamics on the manifold $\{\dot{s}_{\mathbf{i}} \equiv 0\}$, which are given by

$$\dot{\chi}_{\mathbf{i}} = \frac{1}{\Xi_{\mathbf{i}}(t)} \sum_{\mathbf{j}\in\mathcal{N}_{\mathbf{i}}} a_{\mathbf{ij}}(t)\left[\dot{\chi}_{\mathbf{j}} - (\chi_{\mathbf{i}} - \chi_{\mathbf{j}})\right], \tag{32}$$

where $\chi_{\mathbf{i}} = \dot{x}_{\mathbf{i}} + \alpha x_{\mathbf{i}}$. It follows from Lemma 1 that $\chi_{\mathbf{i}}(t) - \chi_{\mathbf{j}}(t) \to 0$ as $t \to +\infty$, for all $\mathbf{i}, \mathbf{j} \in \mathcal{V}$. Therefore, we have

$$(\dot{x}_{\mathbf{i}} - \dot{x}_{\mathbf{j}}) = -\alpha(x_{\mathbf{i}} - x_{\mathbf{j}}) + \epsilon_t, \quad \forall \mathbf{i}, \mathbf{j} \in \mathcal{V}, \tag{33}$$

where $\epsilon_t \to 0$, proving $x_{\mathbf{i}} - x_{\mathbf{j}} \to 0$ as $t \to \infty$. It is clear that, if the communication topology is fixed, i.e., $a_{\mathbf{ij}}(t) \equiv a_{\mathbf{ij}}$ for all $\mathbf{i}, \mathbf{j} \in \mathcal{V}$, then the control law (31) reduces to the Slotine-Li controller (29) when $\alpha = 0$.

This idea can be generalized to the m-th order integrator-chain model. The generalized Slotine-Li controller is given by

$$\begin{aligned} z_{\mathbf{i}}^{(m-1)} &= \frac{1}{\Xi_{\mathbf{i}}(t)} \sum_{j\in\mathcal{N}_{\mathbf{i}}(t)} a_{\mathbf{ij}}(t)\Big[x_{\mathbf{j}}^{(m)} - (1+\alpha_{m-1})\,\Delta_{\mathbf{ij}}^{(m-1)} \\ &\quad - (\alpha_{m-1}+\alpha_{m-2})\,\Delta_{\mathbf{ij}}^{(m-2)} - \cdots - (\alpha_2+\alpha_1)\dot{\Delta}_{\mathbf{ij}} - \alpha_1\Delta_{\mathbf{ij}}\Big] \\ s_{\mathbf{i}} &= \dot{x}_{\mathbf{i}} - z_{\mathbf{i}}, \\ u_{\mathbf{i}} &= z_{\mathbf{i}}^{(m-1)} - k_{1\mathbf{i}}s_{\mathbf{i}} - \cdots - k_{(m-1)\mathbf{i}}s_{\mathbf{i}}^{(m-2)}, \end{aligned} \tag{34}$$

where $\Delta_{\mathbf{ij}} = x_{\mathbf{i}} - x_{\mathbf{j}}$; the parameters $\alpha_1, \ldots, \alpha_{m-1}$ and $k_{1\mathbf{i}}, \ldots, k_{(m-1)\mathbf{i}}$ are chosen such that the matrix $A(\alpha_1, \ldots, \alpha_{m-1})$ and $A\left(k_{1\mathbf{i}}, \ldots, k_{(m-1)\mathbf{i}}\right)$ are Hurwitz, respectively. The matrix $A(\cdot)$ is defined as

$$A(k_1, \ldots, k_{m-1}) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -k_1 & -k_2 & -k_3 & \cdots & -k_{m-1} \end{bmatrix}.$$

Theorem 1. *Consider the m-th order integrator-chain model $x_{\mathbf{i}}^{(m)} = u_{\mathbf{i}}$, where $x_{\mathbf{i}} \in \mathbb{R}^n$, $\mathbf{i} = 1, \ldots, N$, and $m, n, N \in \mathbb{Z}_{>0}$. Then, under the generalized Slotine-Li controller (34), the consensus tracking problem is solved provided that Assumption 1 holds.*

Proof. Noting that the reference velocity $z_{\mathbf{i}}(t)$ is differentiable up to $(m-1)$-th order, we have

$$s_{\mathbf{i}}^{(m-1)} = x_{\mathbf{i}}^{(m)} - z_{\mathbf{i}}^{(m-1)}. \tag{35}$$

Substituting $x_{\mathbf{i}}^{(m)} = u_{\mathbf{i}}$ into (35) yields $s_{\mathbf{i}}^{(m-1)} = -k_{1\mathbf{i}} s_{\mathbf{i}} - \cdots - k_{(m-1)\mathbf{i}} s_{\mathbf{i}}^{(m-2)}$. The condition $A\left(k_{1\mathbf{i}}, \ldots, k_{(m-1)\mathbf{i}}\right)$ being Hurwitz implies that $\left(s_{\mathbf{i}}, \dot{s}_{\mathbf{i}}, \ldots, s_{\mathbf{i}}^{(m-1)}\right)(t) \to 0$ exponentially as $t \to +\infty$. On the other hand, substituting $z_{\mathbf{i}}^{(m-1)}$ into (35), and denoting $q_{\mathbf{i}} = x_{\mathbf{i}}^{(m-1)} + \alpha_{(m-1)} x_{\mathbf{i}}^{(m-2)} + \cdots + \alpha_2 \dot{x}_{\mathbf{i}} + \alpha_1 x_{\mathbf{i}}$, we recover the first-order consensus algorithm

$$\dot{q}_{\mathbf{i}} = \frac{1}{\Xi_{\mathbf{i}}(t)} \sum_{\mathbf{j} \in \mathcal{N}_{\mathbf{i}}(t)} a_{\mathbf{ij}}(t) \left[\dot{q}_{\mathbf{j}} - (q_{\mathbf{i}} - q_{\mathbf{j}})\right] + s_{\mathbf{i}}^{(m-1)}(t), \tag{36}$$

which can be viewed as an exponentially stable linear system (with respect to the equilibrium manifold $\{(x_{\mathbf{1}}, \ldots, x_{\mathbf{N}}) : x_{\mathbf{i}} = x_{\mathbf{j}}, \forall \mathbf{i}, \mathbf{j} \in \mathcal{V}\}$) with an exponentially decaying input $s_{\mathbf{i}}^{(m-1)}(t)$. It follows from Lemma 1 and the converging-input converging-state (CICS) property for stable linear systems that the exponential consensus is achieved for variable $q_{\mathbf{i}}$. That is, $|q_{\mathbf{i}}(t) - q_{\mathbf{j}}(t)| \to 0$ exponentially as $t \to +\infty$, for all $\mathbf{i}, \mathbf{j} \in \mathcal{V}$. Finally, it follows from the condition $A(\alpha_1, \ldots, \alpha_{m-1})$ being Hurwitz that $|x_{\mathbf{i}}(t) - x_{\mathbf{j}}(t)| \to 0$ exponentially as $t \to +\infty$, for all $\mathbf{i}, \mathbf{j} \in \mathcal{V}$, where the consensus tracking problem is solved. □

We apply the proposed design to the formation control problem for heterogeneous spatial underactuated vehicle networks.

Position control design. We propose the following observer-based generalized Slotine-Li control law for $\nu_{\mathbf{i}}$

$$\begin{aligned} \dot{\mathfrak{z}}_{1\mathbf{i}} &= \frac{1}{\Xi_{\mathbf{i}}(t)} \sum_{\mathbf{j} \in \mathcal{N}_{\mathbf{i}}(t)} a_{\mathbf{ij}}(t)[\ddot{\xi}_{\mathbf{j}} - (\alpha_1 + 1)(\dot{\xi}_{\mathbf{i}} - \dot{\xi}_{\mathbf{j}}). \\ &\quad + \alpha_1 (R(\eta_{\mathbf{i}}) \hat{\zeta}_{\mathbf{ij}} + d_{\mathbf{ij}})], \\ s_{1\mathbf{i}} &= \dot{\xi}_{\mathbf{i}} - \mathfrak{z}_{1\mathbf{i}}, \\ \nu_{\mathbf{i}} &= \dot{\mathfrak{z}}_{1\mathbf{i}} - k_{1\mathbf{i}} s_{1\mathbf{i}}, \end{aligned} \tag{37}$$

where $\alpha_1, k_{1\mathbf{i}} > 0$ are the control gains; $\hat{\zeta}_{\mathbf{ij}}(t)$ is the output of the sliding mode observer (14)-(15).

Attitude control design. It is clear that the attitude dynamics (4) are decoupled and controlled by three independent control inputs, i.e., $\tilde{\tau}_{\mathbf{i}} = [\tilde{\tau}_{\phi\mathbf{i}}, \tilde{\tau}_{\theta\mathbf{i}}, \tilde{\tau}_{\psi\mathbf{i}}]^\top$. For the three cases discussed in Section 4.1, we apply the generalized Slotine-Li control law to the independently controlled attitude variable. Specifically, for **Case 1**, we propose

$$\begin{aligned} \dot{\mathfrak{z}}_{2\mathbf{i}} &= \frac{1}{\Xi_{\mathbf{i}}(t)} \sum_{\mathbf{j} \in \mathcal{N}_{\mathbf{i}}(t)} a_{\mathbf{ij}}(t)[\ddot{\phi}_{\mathbf{j}} - (\alpha_2 + 1)(\dot{\phi}_{\mathbf{i}} - \dot{\phi}_{\mathbf{j}}) \\ &\quad - \alpha_2 (\phi_{\mathbf{i}} - \phi_{\mathbf{j}})], \\ s_{2\mathbf{i}} &= \dot{\xi}_{\mathbf{i}} - \mathfrak{z}_{2\mathbf{i}}, \\ \tilde{\tau}_{\phi\mathbf{i}} &= \dot{\mathfrak{z}}_{2\mathbf{i}} - k_{2\mathbf{i}} s_{2\mathbf{i}}, \end{aligned} \tag{38}$$

where $\alpha_2, k_{2\mathbf{i}} > 0$ are the control gains. Then, the thrust $u_{\mathbf{i}}(t)$ and the other two desired attitude signals $(\theta_{\mathbf{i}d}(t), \psi_{\mathbf{i}d}(t))$ are given by (20)-(22). We choose the sliding mode control for double-integrator $(\theta_{\mathbf{i}}, \psi_{\mathbf{i}})$-subsystems because of its simplicity and robustness:

$$\tilde{\tau}_{\theta\mathbf{i}} = -\lambda_1 \dot{\tilde{\theta}}_{\mathbf{i}} - k_{3\mathbf{i}} \operatorname{sign}(s_{3\mathbf{i}}), \quad s_{3\mathbf{i}} = \dot{\tilde{\theta}}_{\mathbf{i}} + \lambda_1 \tilde{\theta}_{\mathbf{i}}, \tag{39}$$

$$\tilde{\tau}_{\psi\mathbf{i}} = -\lambda_2 \dot{\tilde{\psi}}_{\mathbf{i}} - k_{4\mathbf{i}} \operatorname{sign}(s_{4\mathbf{i}}), \quad s_{4\mathbf{i}} = \dot{\tilde{\psi}}_{\mathbf{i}} + \lambda_2 \tilde{\psi}_{\mathbf{i}}, \tag{40}$$

where $\tilde{\theta}_{\mathbf{i}} = \theta_{\mathbf{i}} - \theta_{\mathbf{i}d}$; $\tilde{\psi}_{\mathbf{i}} = \psi_{\mathbf{i}} - \psi_{\mathbf{i}d}$; and $\lambda_1 > 0$, $\lambda_2 > 0$, $k_{3\mathbf{i}} > \sup\{|\ddot{\theta}_{\mathbf{i}d}(t)|\}$, and $k_{4\mathbf{i}} > \sup\{|\ddot{\psi}_{\mathbf{i}d}(t)|\}$ are control gains. For **Case 2** and **Case 3**, replace the independently controlled attitude variable ϕ in (38) by θ and ψ, respectively; generate thrust and desired attitude signals using (23)-(25) and (26)-(28), respectively; and replace (θ, ψ) in (39)-(40) by (ϕ, ψ) and (ϕ, θ), respectively.

Theorem 2. *Consider the vehicle dynamics (19), (4). Suppose that Assumption 1 holds. Then, the controller (37)-(40), together with the finite-time sliding mode observer (14)-(15), solves the formation tracking problem.*

Proof. The vehicle dynamics (19), (4) is in the cascaded structure. First, for the $\eta_{\mathbf{i}}$-subsystem, substituting (39)-(40) into (4), yields $s_{3\mathbf{i}}(t) \to 0$ and $s_{4\mathbf{i}}(t) \to 0$ in finite time. On the sliding manifolds $\{s_{3\mathbf{i}} = 0\}$ and $\{s_{4\mathbf{i}} = 0\}$, we have $\theta_{\mathbf{i}}(t) - \theta_{\mathbf{i}d}(t) \to 0$ and $\psi_{\mathbf{i}}(t) - \psi_{\mathbf{i}d}(t) \to 0$ exponentially as $t \to +\infty$. We conclude that $|\eta_{\mathbf{i}}(t) - \eta_{\mathbf{i}d}(t)| \to 0$ exponentially, and thus, the interconnection term $g_{\mathbf{i}}(\eta_{\mathbf{i}}(t), u_{\mathbf{i}}(t), \dot{\xi}_{\mathbf{i}}(t), \nu_{\mathbf{i}}(t)) \to 0$ as $t \to +\infty$. The same conclusion can be easily obtained for **Case 2** and **Case 3**. Also note that the controller (38) is exact the generalized Slotine-Li controller (31). We conclude that $\phi_{\mathbf{i}}(t) - \phi_{\mathbf{j}}(t) \to 0$ as $t \to +\infty$.

Then, it follows from Proposition 1 that, after a finite time T_r, $\hat{\zeta}_{\mathbf{ij}}(t) \equiv \zeta_{\mathbf{ij}}(t) \equiv R(\eta_{\mathbf{i}})^\top (\xi_{\mathbf{j}} - \xi_{\mathbf{i}})$. Substitute into (37), which recovers the generalized Slotine-Li controller structure (31) again. Note that $\dot{s}_{1\mathbf{i}} = \ddot{\xi}_{\mathbf{i}} - \dot{\mathfrak{z}}_{1\mathbf{i}} = -k_{1\mathbf{i}} s_{1\mathbf{i}} + g_{\mathbf{i}}(\eta_{\mathbf{i}}(t), u_{\mathbf{i}}(t), \dot{\xi}_{\mathbf{i}}(t), \nu_{\mathbf{i}}(t))$, and the last term $g_{\mathbf{i}}(\eta_{\mathbf{i}}(t), u_{\mathbf{i}}(t), \dot{\xi}_{\mathbf{i}}(t), \nu_{\mathbf{i}}(t)) \to 0$ as $t \to +\infty$. The CICS property of linear systems implies that $s_{1\mathbf{i}}(t) \to 0$. Finally, it follows from the proof of Theorem 1 that the control objective (5) is achieved, which completes the proof. □

5. NUMERICAL SIMULATION

In this section, we apply the proposed formation control strategy to a heterogeneous spatial underactuated vehicle network including one AUV and four quadrotor unmanned aerial vehicles (UAVs). We number the four quadrotors from **1** to **4**, and the AUV **5**. Assume that the desired formation shape of the group of quadrotors is a horizontal square. Length of the square sides is 5 m. The desired XY position of the AUV **5** is the center of the square in the

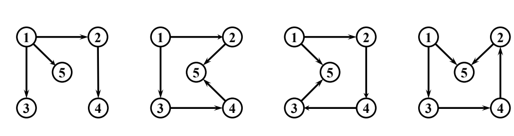
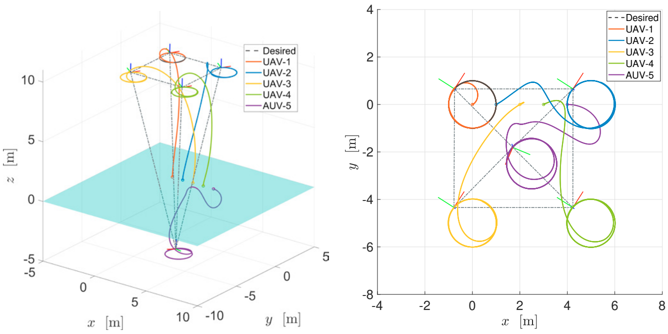
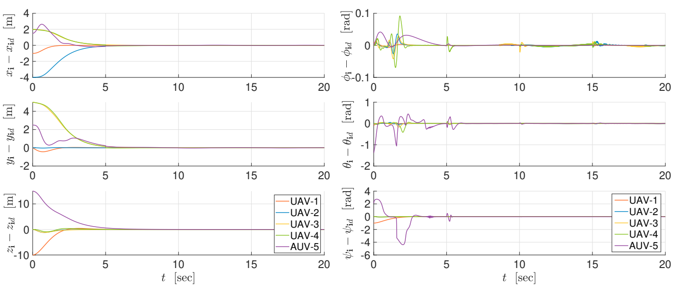


Fig. 2. Directed switching topologies.



(a) (b)

Fig. 3. The trajectories of the five vehicles.



(a) (b)

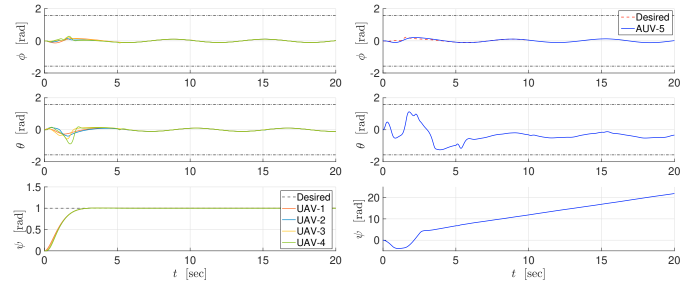
Fig. 4. The configuration errors of the five vehicles.

formation, and the vertical position is 15 m lower than the horizontal square. In the simulation, the group leader is commanded to follow a circle of radius 1 m centered at $(0, 0, 10)$ and at a constant speed of 1 rad/s. The desired yaw angle for the leader vehicle is 1 rad. The quadrotor parameters are selected as: $m_{\mathbf{i}} = 1$ kg, $D_{v\mathbf{i}} = \text{diag}\{0, 0, 0\}$ for $\mathbf{i} = \mathbf{1}, \ldots, \mathbf{4}$. The AUV parameters are selected as: $m_{\mathbf{5}} = 11.85$ kg, $D_{v\mathbf{5}} = \text{diag}\{0.85, 3.11, 0.24\}$. The buoyancy force of the AUV is 114.2 N. All vehicles start from rest and the initial Euler angles are 0. The directed communication graph $\mathcal{G}(t)$ switches every 5 seconds from $\mathcal{G}_{(1)}$ to $\mathcal{G}_{(4)}$, as shown in Fig. 2. The components of the adjacency matrix are $a_{\mathbf{ij}}(t) = 1$ if $(\mathbf{j}, \mathbf{i}) \in \mathcal{E}(t)$. We select $\alpha = 5$ and $\gamma = 5$. The control gains for the four quadrotors are selected as: $k_{1\mathbf{i}} = k_{2\mathbf{i}} = 3$, $k_{3\mathbf{i}} = k_{4\mathbf{i}} = 20$, $\alpha_1 = 2$, $\alpha_2 = 3$, $\lambda_1 = \lambda_2 = 3$. The control gains for the AUV are selected as: $k_{1\mathbf{5}} = 2$, $k_{2\mathbf{5}} = 1$, $k_{3\mathbf{5}} = k_{4\mathbf{5}} = 25$, $\alpha_1 = \alpha_2 = 1$, $\lambda_1 = \lambda_2 = 5$.

Simulation results are illustrated in Figs. 3-5. Figure 3 shows the paths of all five vehicles with the formation illustrated at $t = 20$ s. Figure 4 shows the configuration errors of the five vehicles in the formation. Figure 5 shows the Euler angles of the five vehicles in the formation. The yaw angles of all four quadrotors are in consensus and converge to 1 rad. The roll angle of the AUV converges to the desired trajectory assigned by the quadrotors, while its yaw angle linearly increases as time tends to infinity.

6. CONCLUSIONS

The formation control problem for a team of heterogeneous spatial underactuated vehicles subject to switching topologies has been addressed. A distributed sliding mode observer is used to estimate ranges between vehicles in finite time. Then, the generalized Slotine-Li controller is presented to deal with switching topologies. Global asymptotic convergence is proved for the closed-loop system based on cascaded structure of the vehicle systems.



(a) (b)

Fig. 5. Euler angles of the five vehicles in the formation.

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