Proceedings of the ASME 2020 Dynamic Systems and Control Conference
DSCC 2020
October 4-7, 2020, Pittsburgh, USA

DSCC2020-3178

FORMATION CONTROL FOR UNDERACTUATED SURFACE VESSEL NETWORKS

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ABSTRACT

Developing distributed control algorithms for multi-agent systems is difficult when each agent is modeled as a nonlinear dynamical system. Moreover, the problem becomes far more complex if the agents do not have sufficient number of actuators to track any arbitrary trajectory. In this paper, we present the first fully decentralized approach to formation control for networks of underactuated surface vessels. The vessels are modeled as three degree of freedom planar rigid bodies with two actuators. Algebraic graph theory is used to model the network as a directed graph and employing a leader-follower model. We take advantage of the cascade structure of the combined nonlinear kinematic and dynamic model of surface vessels and develop a reduced-order error dynamic model using a state transformation definition. The error dynamics and consequently all system states are then stabilized using sliding mode control approach. It is shown that the stabilization of the reduced-order error dynamics guarantees uniform global asymptotic stability of the closed-loop system subject to bounded uncertainties. The proposed control method can be implemented in directed time-invariant communication networks without the availability of global position measurements for any of the vehicles participating in the network. An example of a a network of five surface vessels is simulated to verify the effective performance of the proposed control approach.

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INTRODUCTION

In this paper, we study the underactuated formation control problems of surface vessel networks. For a single vessel, it is common to consider the motion in a horizontal plane, where the motion in heave, roll and pitch are neglected. The vessel then is assumed to be controlled by only two propellers, which are the force in surge and the control torque in yaw. Over the past twenty years, significant efforts have been devoted to handle the control problems of a single underactuated surface vessel. Such problems include solving the set-point regulation problem using time-varying approach [1, 2], discontinuous control technique [3, 4]; solving the trajectory tracking problem using Lyapunov function approach [5, 6], sliding mode approach [7, 8] and backstepping design [9]; and solving the path following problem using adaptive control strategy [10].

Trajectory tracking decentralized control of networks using leader-follower formation can be considered as classical trajectory tracking control problem extended to the multi-agent systems. Research on cooperative control of surface vessel networks has grown overwhelmingly over the past few years since it has many applications in practice such as reconnaissance, marine search and rescue missions, and mine clearance, to name a few. Various approaches to cooperative control have been proposed, for example, using leader-follower strategy [11, 12], behavior-based method [13, 14] and virtual structure approach [15, 16]. In particular, several formation control approaches were proposed for underactuated surface vessel networks. In [17], a sliding mode formation control design was proposed for underactuated surface vessel networks. However, it is assumed that in [17] the trajectory of the leader agent can be obtained in real-time by all the group members of the network, which is not a practical assumption in a distributed network structure. In [18, 19], cooperative control protocols were designed for underactuated surface vessels to achieve static formation and formation-tracking tasks, respectively. However, in these control designs, agents sense their positions with respect to the global coordinate system and interactions are not necessarily required because the desired formation can be achieved by position control of individual agents. In [20], a finite-time formation control protocol was proposed for surface vessels using sliding mode technique. However the proposed control protocol is a three-dimensional vector, which is difficult to implement for underactuated systems in practice. In [21], cooperative control laws were presented for underactuated vessels with limited sensing ranges to perform a desired formation, and guarantee no collisions between the vessels. However, in this design, the control laws also require the global position measurements of the follower vessels.

Unlike centralized controlled where every agent must have global information and awareness, in decentralized control, each agent can only obtain the local relative position information between the agent itself and its neighbors. Therefore, control design using the local relative position information is more reliable. In this paper, we present a novel leader-follower formation tracking framework for networks of underactuated surface vessels without measurements of global positions. By exploiting the cascade structure of the vessel model, a transformation is proposed to reduce the order of error dynamics. Then, a sliding mode control design is employed to stabilize the reduced-order error dynamics and consequently uniform global asymptotic stability is established for the closed-loop system under bounded uncertainties. Numerical simulations are presented to illustrate the effectiveness and robustness of the proposed approaches. This work represents the first fully decentralized approach to formation control of networks of underactuated surface vessels.

PROBLEM FORMULATION

Surface Vessel Model

Consider a network of $N+1$ homogeneous planar underactuated surface vessels where the vessels are numbered $\mathbf{i} = 0,1,\ldots,N$ with 0 representing a real or virtual leader, and $1,\ldots,N$ are the follower vessels. Each vessel considered in the network is assumed to have only two actuators, which provide surge control force and a yaw control moment and its heave, roll and pitch displacements are neglected. The motion of a single vessel $\mathbf{i}$ in the network is described by assigning a body-fixed reference frame $\{x_{b\mathbf{i}} y_{b\mathbf{i}}\}$ to its center of mass located at $[x_\mathbf{i}, y_\mathbf{i}]^\top \in \mathbb{R}^2$ with respect to a fixed inertial reference frame $\{XY\}$ and yaw angle $\theta_\mathbf{i} \in \mathbb{R}$, as shown in Fig. 1. The vessel model is represented by its nonholonomic kinematic relations and its equations of motion as follows

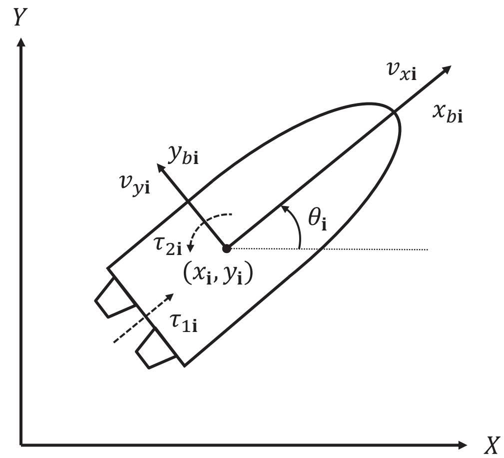

FIGURE 1: Top view of the underactuated surface vessel $\mathbf{i}$.

$$
\begin{aligned}
\dot{x}_\mathbf{i} &= v_{x\mathbf{i}} \cos\theta_\mathbf{i} - v_{y\mathbf{i}} \sin\theta_\mathbf{i} \\
\dot{y}_\mathbf{i} &= v_{x\mathbf{i}} \sin\theta_\mathbf{i} + v_{y\mathbf{i}} \cos\theta_\mathbf{i} \\
\dot{\theta}_\mathbf{i} &= \omega_\mathbf{i} \\
\dot{v}_{x\mathbf{i}} &= \frac{m_{22}}{m_{11}} v_{y\mathbf{i}} \omega_\mathbf{i} - \frac{d_{11}}{m_{11}} v_{x\mathbf{i}} + \delta_{x\mathbf{i}}(t) + \tau_{1\mathbf{i}} \\
\dot{v}_{y\mathbf{i}} &= -\frac{m_{11}}{m_{22}} v_{x\mathbf{i}} \omega_\mathbf{i} - \frac{d_{22}}{m_{22}} v_{y\mathbf{i}} + \delta_{y\mathbf{i}}(t) \\
\dot{\omega}_\mathbf{i} &= \frac{m_{11} - m_{22}}{m_{33}} v_{x\mathbf{i}} v_{y\mathbf{i}} - \frac{d_{33}}{m_{33}} \omega_\mathbf{i} + \delta_{\omega\mathbf{i}}(t) + \tau_{2\mathbf{i}},
\end{aligned}
\tag{1}
$$

where $[v_{x\mathbf{i}}, v_{y\mathbf{i}}]^\top \in \mathbb{R}^2$ represent the velocity of the vessel center of mass in the body-fixed frame $\{x_{b\mathbf{i}} y_{b\mathbf{i}}\}$ and $\omega_\mathbf{i} \in \mathbb{R}$ is its angular velocity. The parameters m_{kk} and d_{kk}, $k = 1,2,3$ are the inertia and damping parameters which are positive and assumed to be constant. Note that for marine vehicle models $m_{11} \neq m_{22}$ due to added mass effect. The terms $\delta_{x\mathbf{i}}(t), \delta_{y\mathbf{i}}(t), \delta_{\omega\mathbf{i}}(t)$ represent the unknown but bounded modeling uncertainties and disturbances, i.e., $|\delta_{x\mathbf{i}}(t)| \leq \Delta_{x\mathbf{i}}, |\delta_{y\mathbf{i}}(t)| \leq \Delta_{y\mathbf{i}}, |\delta_{\omega\mathbf{i}}(t)| \leq \Delta_{\omega\mathbf{i}}$, where $\Delta_{x\mathbf{i}}, \Delta_{y\mathbf{i}}, \Delta_{\omega\mathbf{i}}$ are known positive constants; $\tau_{1\mathbf{i}}$ and $\tau_{2\mathbf{i}}$ are the scaled control inputs representing the surge force and yaw moment, respectively.

Algebraic Graph Theory

We use graph theory to define the communication network. Network topology of the $N+1$ surface vessels is defined by a directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ where $\mathscr{V}$ and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ represent its sets of vertices and edges, respectively. There are $N+1$ nodes whose dynamics are described in (1). The set of neighboring nodes with edges connected to node $\mathbf{i}$ is denoted by

$\Omega_{\mathbf{i}} = \{\mathbf{j} \mid (\mathbf{j},\mathbf{i}) \in \mathscr{E}\}$. The edges represent communication between the nodes such that node $\mathbf{j}$ can obtain information from node $\mathbf{i}$ for feedback control purposes, if $\mathbf{j} \in \Omega_{\mathbf{i}}$. In order to incorporate a combination of neighboring feedback information from neighboring nodes, we let $w_{\mathbf{ij}} \geq 0$ be a weighing factor for any $\mathbf{i},\mathbf{j} \in \mathscr{V}$. These factors are selected such that $\sum_{\mathbf{j}\in\Omega_{\mathbf{i}}} w_{\mathbf{ij}} = 1$ if $(\mathbf{j},\mathbf{i}) \in \mathscr{E}$ and $w_{\mathbf{ij}} = 0$, otherwise. We assume that there exists at least one directed path starting from the leader to any other node in the network and that the communication topology is time invariant. Furthermore, there are no loops in the graph and the leader does not receive any communication from other nodes.

Feasible Reference Trajectories

The reference trajectory for all three pose states of surface vessels cannot be arbitrarily selected due to underactuation. Hence, we must define feasible reference trajectories relying on the vessel model. Since we are considering formation control of networks of homogeneous vessels, then the most important step is to generate a feasible reference trajectory for the leader vessel $\mathbf{0}$, irrespective of it being real or virtual. Since tracking position is essential in any surface vessel problem, then we assume the leader must follow a smooth reference trajectory specified by $[x_{\mathbf{0}}(t), y_{\mathbf{0}}(t)]^\top$. Then, as proposed in [22], the feasible reference trajectory for $\theta_{\mathbf{0}}(t)$ can be calculated by integrating the nonholonomic constraint in (1) subject to the initial condition $\theta_{\mathbf{0}}(0) = \theta_{\mathbf{0},0}$:

$$\dot{v}_{y\mathbf{0}}(t) = -\frac{m_{11}}{m_{22}} v_{x\mathbf{0}}(t)\dot{\theta}_{\mathbf{0}}(t) - \frac{d_{22}}{m_{22}} v_{y\mathbf{0}}(t), \tag{2}$$

Equation (2) is a first order differential equation in $\theta_{\mathbf{0}}(t)$ where

$$\begin{aligned} v_{x\mathbf{0}}(t) &= \cos\theta_{\mathbf{0}}(t)\dot{x}_{\mathbf{0}}(t) + \sin\theta_{\mathbf{0}}(t)\dot{y}_{\mathbf{0}}(t) \\ v_{y\mathbf{0}}(t) &= -\sin\theta_{\mathbf{0}}(t)\dot{x}_{\mathbf{0}}(t) + \cos\theta_{\mathbf{0}}(t)\dot{y}_{\mathbf{0}}(t) \\ \dot{v}_{y\mathbf{0}}(t) &= -\sin\theta_{\mathbf{0}}(t)\ddot{x}_{\mathbf{0}}(t) + \cos\theta_{\mathbf{0}}(t)\ddot{y}_{\mathbf{0}}(t) - v_{x\mathbf{0}}(t)\dot{\theta}_{\mathbf{0}}(t) \end{aligned} \tag{3}$$

Note that if the leader is virtual, then $[x_{\mathbf{0}}(t), y_{\mathbf{0}}(t)]^\top$ and $\theta_{\mathbf{0}}(t)$ derived from (2) are sufficient in describing its motion. However, if the leader is a real vessel, then the control law presented in [22] may be employed to follow the reference trajectory.

As far as all the follower vessels are concerned, their feasible trajectories are dictated by the formation. For these agents, the position trajectory is determined by the desired formation. Since the leader is following a feasible trajectory, the followers can track any position relative to their neighbors, while relative orientations are dictated by those neighbors. For example, when the overall formation is non-rotating, the vessel relative orientations must be zero.

ERROR DYNAMICS

Let us define the set $\ell_{\mathbf{ij}} \in \mathbb{R}^3$, $\mathbf{i} = 1,\ldots,N$, $\mathbf{j} \in \Omega_{\mathbf{i}}$ of relative poses of the agents that describes the desired formation of the network. The desired formation is basically a geometric pattern and, as explained in previous section, we set the third component $\ell^3_{\mathbf{ij}} = 0$ to achieve feasible trajectory. Denoting the configuration of agent $\mathbf{i}$ by $q_{\mathbf{i}} = [x_{\mathbf{i}}, y_{\mathbf{i}}, \theta_{\mathbf{i}}]^\top$, the formation tracking errors can be defined by

$$\varepsilon_{\mathbf{ij}} = q_{\mathbf{i}} - q_{\mathbf{j}} - \ell_{\mathbf{ij}}, \ \forall\, \mathbf{i} = 1,\ldots,N, \ \forall\, \mathbf{j} \in \Omega_{\mathbf{i}}, \tag{4}$$

Next, we redefine the formation tracking errors for agents $\mathbf{i} = 1,\ldots,N$ using the following transformation $z_{\mathbf{i}} = [z_{1\mathbf{i}}, z_{2\mathbf{i}}, z_{3\mathbf{i}}]^\top$:

$$z_{\mathbf{i}} = R(\theta_{\mathbf{i}})^\top \left(\dot{\tilde{q}}_{\mathbf{i}} + \Lambda\tilde{q}_{\mathbf{i}}\right), \tag{5}$$

where $\tilde{q}_{\mathbf{i}} := q_{\mathbf{i}} - \sum_{j\in\Omega_{\mathbf{i}}} w_{\mathbf{ij}}\left(q_{\mathbf{j}} + \ell_{\mathbf{ij}}\right)$, $R(\theta_{\mathbf{i}})$ is the orthogonal rotation matrix

$$R(\theta_{\mathbf{i}}) := \begin{bmatrix} \cos\theta_{\mathbf{i}} & -\sin\theta_{\mathbf{i}} & 0 \\ \sin\theta_{\mathbf{i}} & \cos\theta_{\mathbf{i}} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{6}$$

and $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \lambda_3\} > 0$ is a diagonal positive definite matrix.

Since $R(\theta_{\mathbf{i}})$ is orthogonal matrix, $z_{\mathbf{i}} \to 0$ implies $\left(\dot{\tilde{q}}_{\mathbf{i}} + \Lambda\tilde{q}_{\mathbf{i}}\right) \to 0$, which may be written as

$$\dot{\tilde{q}}_{\mathbf{i}} = -\Lambda\tilde{q}_{\mathbf{i}} + \zeta_{\mathbf{i}}(t), \quad \lim_{t\to\infty} \zeta_{\mathbf{i}}(t) = 0, \tag{7}$$

where $\zeta_{\mathbf{i}}(t) = R(\theta_{\mathbf{i}})z_{\mathbf{i}}$. Since $\Lambda > 0$, then by the converging-input-converging-state (CICS) property of stable linear systems [23], we conclude that $q_{\mathbf{i}} - \sum_{j\in\Omega_{\mathbf{i}}} w_{\mathbf{ij}}\left(q_{\mathbf{j}} + \ell_{\mathbf{ij}}\right) \to 0$ as $t \to \infty$. Since this convergence holds for all $\mathbf{i} = 1,\ldots,N$, we can conclude that $\varepsilon_{\mathbf{ij}} \to 0$, $\forall\, \mathbf{j} \in \Omega_{\mathbf{i}}$ [11]. Moreover, the formation error for all agents in the network converge to zero [24].

The objective of the decentralized formation control design is therefore to drive $z_{\mathbf{i}}$, $\mathbf{i} = 1,\ldots,N$ to zero asymptotically. The error dynamic model in terms of $z_{\mathbf{i}}$ is determined by taking the time derivative of (5):

$$\dot{z}_{\mathbf{i}} = \dot{J}(\theta_{\mathbf{i}})^\top \left(\dot{\tilde{q}}_{\mathbf{i}} + \Lambda\tilde{q}_{\mathbf{i}}\right) + R(\theta_{\mathbf{i}})^\top \left(\ddot{\tilde{q}}_{\mathbf{i}} + \Lambda\dot{\tilde{q}}_{\mathbf{i}}\right) \tag{8}$$

Substituting equations in (1) into (8), we derive the error dynamic model in a simple reduced form

$$\dot{z}_{\mathbf{i}} = \begin{bmatrix} \omega_{\mathbf{i}} z_{2\mathbf{i}} \\ -\omega_{\mathbf{i}} z_{1\mathbf{i}} \\ 0 \end{bmatrix} + \begin{bmatrix} u_{1\mathbf{i}} \\ \Psi_{\mathbf{i}} \\ u_{2\mathbf{i}} \end{bmatrix} + \begin{bmatrix} \delta_{x\mathbf{i}} \\ \delta_{y\mathbf{i}} \\ \delta_{\omega\mathbf{i}} \end{bmatrix}, \tag{9}$$

where we have used the feedback transformation

$$\begin{aligned}\tau_{1\mathbf{i}} = &\sum_{j\in\Omega_\mathbf{i}} w_{\mathbf{ij}}\left[(\dot v_{x\mathbf{j}} - v_{y\mathbf{j}}\omega_\mathbf{j})\cos(\theta_\mathbf{i}-\theta_\mathbf{j}) + (\dot v_{y\mathbf{j}} + v_{x\mathbf{j}}\omega_\mathbf{j})\sin(\theta_\mathbf{i}-\theta_\mathbf{j})\right] \\ &- \lambda_1\left[v_{x\mathbf{i}} - \sum_{j\in\Omega_\mathbf{i}} w_{\mathbf{ij}}\left[v_{x\mathbf{j}}\cos(\theta_\mathbf{i}-\theta_\mathbf{j}) + v_{y\mathbf{j}}\sin(\theta_\mathbf{i}-\theta_\mathbf{j})\right]\right] \\ &+ \left(1-\frac{m_{22}}{m_{11}}\right)v_{y\mathbf{i}}\omega_\mathbf{i} + \frac{d_{11}}{m_{11}}v_{x\mathbf{i}} + u_{1\mathbf{i}}, \quad (10)\\ \tau_{2\mathbf{i}} = &\frac{1}{a_{\omega\mathbf{i}}}\left[-\frac{m_{11}-m_{22}}{m_{33}}v_{x\mathbf{i}}v_{y\mathbf{i}} + \frac{d_{33}}{m_{33}}\omega_\mathbf{i}\right. \\ &\left.+\sum_{j\in\Omega_\mathbf{i}} w_{\mathbf{ij}}\dot\omega_\mathbf{j} - \lambda_3\left(\omega_\mathbf{i} - \sum_{j\in\Omega_\mathbf{i}} w_{\mathbf{ij}}\omega_\mathbf{j}\right) + u_{2\mathbf{i}}\right], \quad (11)\end{aligned}$$

and $\Psi_\mathbf{i}$ is given as

$$\begin{aligned}\Psi_\mathbf{i} = &\sum_{j\in\Omega_\mathbf{i}} w_{\mathbf{ij}}\left[(\dot v_{x\mathbf{j}} - v_{y\mathbf{j}}\omega_\mathbf{j})\sin(\theta_\mathbf{i}-\theta_\mathbf{j}) - (\dot v_{y\mathbf{j}} + v_{x\mathbf{j}}\omega_\mathbf{j})\cos(\theta_\mathbf{i}-\theta_\mathbf{j})\right] \\ &+ \lambda_2\left[v_{y\mathbf{i}} - \sum_{j\in\Omega_\mathbf{i}} w_{\mathbf{ij}}\left[-v_{x\mathbf{j}}\sin(\theta_\mathbf{i}-\theta_\mathbf{j}) + v_{y\mathbf{j}}\cos(\theta_\mathbf{i}-\theta_\mathbf{j})\right]\right] \\ &+ \left(1-\frac{m_{11}}{m_{22}}\right)v_{x\mathbf{i}}\omega_\mathbf{i} - \frac{d_{22}}{m_{22}}v_{y\mathbf{i}}. \quad (12)\end{aligned}$$

Thus designing a control law for stabilizing the error dynamics of vessel **i** requires not only relative position and orientation of the neighboring agent(s) **j** but also velocities $v_{x\mathbf{j}}, v_{y\mathbf{j}}, \omega_\mathbf{j}$ and accelerations $\dot v_{x\mathbf{j}}, \dot v_{y\mathbf{j}}, \dot\omega_\mathbf{j}$ of agent **j**, which can typically be measured onboard and communicated to agent **i**.

SLIDING MODE CONTROL DESIGN

The goal of decentralized formation control is to design the inputs $[u_{1\mathbf{i}}, u_{2\mathbf{i}}]^\top$ that result in uniformly asymptotically stable error dynamics (9) for all follower agents $\mathbf{i} = 1,\ldots,N$. Here, we take advantage of the vessel model's cascade structure in (1) using the transformed formation error (5) and the reduced three-state error dynamics (9). Note that the error dynamics (9) is structured such that the orientation error $z_{3\mathbf{i}}$ is decoupled from the positioning error $[z_{1\mathbf{i}}, z_{2\mathbf{i}}]^\top$ and thus can be independently controlled.

Let us first design a sliding mode control law $u_{2\mathbf{i}}$ to stabilize $z_{3\mathbf{i}}$ in the following form

$$u_{2\mathbf{i}} = -k_{2\mathbf{i}}\,\mathrm{sign}(z_{3\mathbf{i}}). \quad (13)$$

where $k_{2\mathbf{i}}$ is a control gain. Define the Lyapunov function candidate as $V_{1\mathbf{i}}(z_{3\mathbf{i}}) = \frac{1}{2}z_{3\mathbf{i}}^2$ and design $u_{2\mathbf{i}}$ that makes its time derivative negative:

$$\begin{aligned}\dot V_{1\mathbf{i}} &= z_{3\mathbf{i}}\left[-k_{2\mathbf{i}}\,\mathrm{sign}(z_{3\mathbf{i}}) + \delta_{\omega\mathbf{i}}\right] \\ &\le (-k_{2\mathbf{i}} + \Delta_{\omega\mathbf{i}})|z_{3\mathbf{i}}| < 0, \quad k_{2\mathbf{i}} > \Delta_{\omega\mathbf{i}}.\end{aligned} \quad (14)$$

Since the reaching condition to $z_{3\mathbf{i}} = 0$ is satisfied, it is implied that the orientation error $z_{3\mathbf{i}}$ converges to zero in a finite time that is $\le z_{3\mathbf{i}}(0)/k_{2\mathbf{i}}$.

Now, let us consider the first two equations in (8) and design $u_{2\mathbf{i}}$ to stabilize the remaining error states $(z_{1\mathbf{i}}, z_{2\mathbf{i}})$. As a first step, we define the sliding variable as

$$s_\mathbf{i}(z_{1\mathbf{i}}, z_{2\mathbf{i}}, t) = -\omega_\mathbf{i} z_{1\mathbf{i}} + \Psi_\mathbf{i} + c_\mathbf{i} z_{2\mathbf{i}}, \quad (15)$$

where $c_\mathbf{i} > 0$ is a control gain. Note that the sliding variable is defined such that on the manifold $s_\mathbf{i} = 0$,

$$\dot z_{2\mathbf{i}} = -c_\mathbf{i} z_{2\mathbf{i}} + \delta_{y\mathbf{i}}(t). \quad (16)$$

Equation (16) implies that the zero dynamics on the sliding surface is globally asymptotically stable and $\{s_\mathbf{i} = 0\}$ is a globally invariant manifold as long as the unmatched uncertainty $\delta_{y\mathbf{i}}(t)$ is a vanishing perturbation with respect to $z_{2\mathbf{i}}$.

Next, define the Lyapunov function $V_{2\mathbf{i}}(s_\mathbf{i}) = \frac{1}{2}s_\mathbf{i}^2$ and take its time derivative along $(z_{1\mathbf{i}}, z_{2\mathbf{i}})$ trajectories:

$$\begin{aligned}\dot V_{2\mathbf{i}} &= s_\mathbf{i}\left[\frac{\partial s_\mathbf{i}}{\partial z_{1\mathbf{i}}}\dot z_{1\mathbf{i}} + \frac{\partial s_\mathbf{i}}{\partial z_{2\mathbf{i}}}\dot z_{2\mathbf{i}} + \frac{\partial s_\mathbf{i}}{\partial t}\right] \\ &= s_\mathbf{i}\left[\frac{\partial s_\mathbf{i}}{\partial z_{1\mathbf{i}}}(\omega_\mathbf{i} z_{2\mathbf{i}} + u_{1\mathbf{i}} + \delta_{x\mathbf{i}}) - \frac{\partial s_\mathbf{i}}{\partial z_{2\mathbf{i}}}\left(\omega_\mathbf{i} z_{1\mathbf{i}} - \Psi_\mathbf{i} - \delta_{y\mathbf{i}}\right) + \frac{\partial s_\mathbf{i}}{\partial t}\right].\end{aligned} \quad (17)$$

Assuming that $\partial s_\mathbf{i}/\partial z_{1\mathbf{i}} \ne 0$, we can then define the equivalent control law as

$$u_{1\mathbf{ie}} = -\omega_\mathbf{i} z_{2\mathbf{i}} - \left(\frac{\partial s_\mathbf{i}}{\partial z_{1\mathbf{i}}}\right)^{-1}\left[\frac{\partial s_\mathbf{i}}{\partial z_{2\mathbf{i}}}(-\omega_\mathbf{i} z_{1\mathbf{i}} + \Psi_\mathbf{i}) + \frac{\partial s_\mathbf{i}}{\partial t}\right]. \quad (18)$$

Adding the discontinuous term to the equivalent control, the sliding mode control law is derived as

$$u_{1\mathbf{i}} = u_{1\mathbf{ie}} - \left(\frac{\partial s_\mathbf{i}}{\partial z_{1\mathbf{i}}}\right)^{-1} k_{1\mathbf{i}}\,\mathrm{sign}(s_\mathbf{i}) \quad (19)$$

To determine the stability condition with respect to bounded uncertainties, we substitute from (19) into (17):

$$\begin{aligned}\dot{V}_{2\mathbf{i}} &= s_{\mathbf{i}}\left[-k_{1\mathbf{i}}\,\mathrm{sign}(s_{\mathbf{i}}) + \frac{\partial s_{\mathbf{i}}}{\partial z_{1\mathbf{i}}}\delta_{x\mathbf{i}} + \frac{\partial s_{\mathbf{i}}}{\partial z_{2\mathbf{i}}}\delta_{y\mathbf{i}}\right] \\ &\le |s_{\mathbf{i}}|\left[-k_{1\mathbf{i}} + \left|\frac{\partial s_{\mathbf{i}}}{\partial z_{1\mathbf{i}}}\right|\Delta_{x\mathbf{i}} + \left|\frac{\partial s_{\mathbf{i}}}{\partial z_{2\mathbf{i}}}\right|\Delta_{y\mathbf{i}}\right].\end{aligned} \tag{20}$$

Thus the robustness gain $k_{1\mathbf{i}}$ must be selected such that

$$k_{1\mathbf{i}} > \left|\frac{\partial s_{\mathbf{i}}}{\partial z_{1\mathbf{i}}}\right|\Delta_{x\mathbf{i}} + \left|\frac{\partial s_{\mathbf{i}}}{\partial z_{2\mathbf{i}}}\right|\Delta_{y\mathbf{i}} \tag{21}$$

to satisfy the reaching condition and guaranteeing that all trajectories of $(z_{1\mathbf{i}}, z_{2\mathbf{i}})$ converge to the sliding surface in finite time.

The uniformly asymptotic stability of system (16) can be established under the assumption that the unmatched disturbance $\delta_{y\mathbf{i}}(t)$ satisfies the vanishing perturbation condition with respect to $z_{2\mathbf{i}}$, i.e., $|\delta_{y\mathbf{i}}(t)| \le \kappa_{\mathbf{i}}|z_{2\mathbf{i}}|$, where $\kappa_{\mathbf{i}}$ is a positive constant [25]. Note that, it is possible to reject non-vanishing and unmatched uncertainties using integral sliding manifold design [26]. It is important to note that, the controllers (10), (11), (13) and (19) are completely decentralized and independent of global position measurements.

It should be noted that the singularity condition $\partial s_{\mathbf{i}}/\partial z_{1\mathbf{i}} \neq 0$ implies that $\omega_{\mathbf{i}} \neq 0$. When $\omega_{\mathbf{i}} = 0$, the $(z_{1\mathbf{i}}, z_{2\mathbf{i}})$-dynamics are uncoupled; i.e. $z_{2\mathbf{i}}$ cannot be controlled by $u_{2\mathbf{i}}$. Under this condition, we may stabilize $z_{1\mathbf{i}}$ only by defining $s_{\mathbf{i}} = z_{1\mathbf{i}}$ and $z_{2\mathbf{i}}$ will remain uncontrolled. From a practical viewpoint, this case happens only when the forward and lateral motions are uncoupled which occurs in a straight line motion. In such cases, we may use a hybrid control law where the controller in (19) is replaced with simple control law derived by selecting $s_{\mathbf{i}} = z_{1\mathbf{i}}$, that is $u_{1\mathbf{i}} = -k_{1\mathbf{i}}\,\mathrm{sign}(s_{\mathbf{i}})$ where $k_{1\mathbf{i}} > \Delta_{x\mathbf{i}}$.

SIMULATION RESULTS

In this section, numerical simulation of a network of surface vessels is presented to validate the performance of the proposed formation control protocol. Consider a group of five homogeneous underactuated surface vessels with the communication topology graph shown in Fig. 2. Since agent **4** has two communication edges, we set the weighting coefficients $w_{42} = w_{43} = 0.5$. While, the remaining coefficients are all set to 1. The desired time-invariant formation shape of the four follower vessels is a square that moves along with the leader vessel **0** located at the center of the square. The length of the square sides is 6 m.

In the simulation, the leader is commanded to follow a circle of radius 0.7 m centered at $(0.2\text{ m}, 1\text{ m})$ and at a constant speed of 1.25 rad/s.

The model parameters of the vessels, initial conditions of the five vessels, and the control gains are given as follows:

$$\begin{aligned}&m_{11} = 1.612\text{ kg}, \quad m_{22} = 1.982\text{ kg}, \quad m_{33} = 1.035\text{ kg.m}^2 \\ &d_{11} = 1.436\text{ kg/s}, \quad d_{22} = 12.61\text{ kg/s}, \quad d_{33} = 0.864\text{ kg.m}^2/\text{s} \\ &q_{\mathbf{0}}(0) = [0,0,0]^\top, \quad q_{\mathbf{1}}(0) = [0,-1\text{ m},\pi/2]^\top, \\ &q_{\mathbf{2}}(0) = [-2\text{ m},2\text{ m},0]^\top, \quad q_{\mathbf{3}}(0) = [3\text{ m},0,0]^\top, \\ &q_{\mathbf{4}}(0) = [-2\text{ m},0,\pi]^\top, \quad \lambda_1 = \lambda_2 = \lambda_3 = 2 \\ &k_{1\mathbf{i}} = k_{2\mathbf{i}} = c_{\mathbf{i}} = 1, \quad \mathbf{i} = 1,2,3,4\end{aligned}$$

The formation errors of the group of vessels are shown in Fig. 3 and the formation path is shown in Fig. 4. As shown in Fig. 3, the errors converge after approximately 8 seconds. The formation of the four follower vessels is also illustrated in Fig. 4, demonstrating that the desired square shape has been achieved.

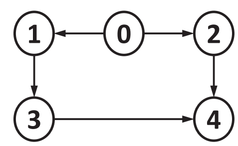

FIGURE 2: Communication topology graph of the five underactuated surface vessels in the network

CONCLUSION

In this work, we presented a decentralized formation control approach for underactuated surface vessels with a given communication topology. The approach was based on leader-follower model relying on relative coordination and thus *without* requiring any global position measurements. Using the cascade structure of vehicle dynamic model, a transformation was proposed to reduce the order of error dynamics, and then a nonlinear *distributed* sliding mode design was employed to stabilize the error dynamics. The control law did not require linearization or simplification of the rigid body dynamics and is shown to be robust with respect to bounded uncertainties and disturbances. An example was numerically simulated to illustrate the effectiveness and robustness of the proposed framework. Our future research will concentrate on time-varying formation control of heterogeneous surface vessels and other forms of vehicles.

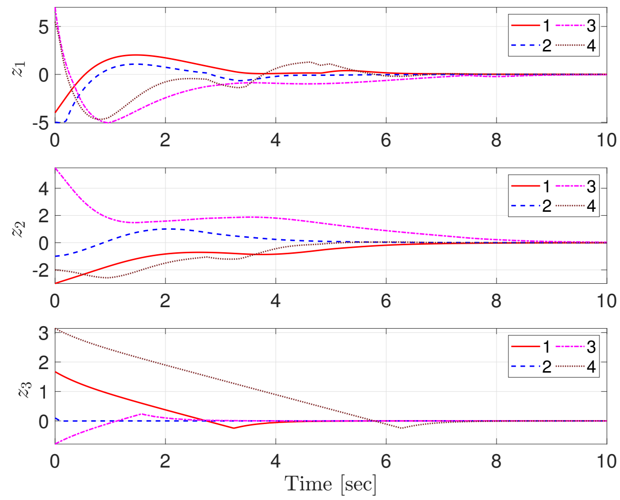

FIGURE 3: Formation errors of the surface vessels in the network

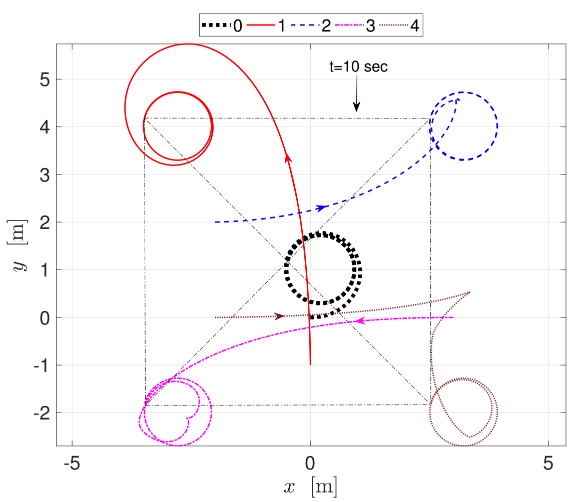

FIGURE 4: Paths of the surface vessels in the network

ACKNOWLEDGMENT

This research was supported in part by the U.S. Office of Naval Research under Grant N00014-19-1-2255.

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