2021 60th IEEE Conference on Decision and Control (CDC)
December 13-15, 2021. Austin, Texas

Leader-Follower Formation Stabilization Control for Planar Underactuated Vehicle Networks

Bo Wang, Hashem Ashrafiuon, and Sergey Nersesov

2021 60th IEEE Conference on Decision and Control (CDC) | 978-1-6654-3659-5/21/$31.00 ©2021 IEEE | DOI: 10.1109/CDC45484.2021.9682979

***Abstract*— In this paper, we solve the distributed leader-follower formation stabilization control problem for generic planar underactuated vehicle networks without global position measurements. The vehicles in the network are modeled as generic 3-DOF planar rigid bodies with two control inputs. By incorporating graph theory, passivity-based control, partial stability theory, Matrosov's theorem and the persistence of excitation concept, a smooth time-varying formation control law is proposed to address the formation stabilization problem. Moreover, the structure of the controller is simple compared to the existing controllers in the literature, and thus, is practical and easy to implement. Simulations on a group of underactuated surface vessels are presented to demonstrate the effectiveness of the proposed control scheme.**

***Index Terms*— Formation stabilization, underactuated systems, persistency of excitation, Lagrangian systems.**

I. Introduction

A mechanical system is underactuated if it has fewer number of independent actuators than its degrees of freedom. Planar vehicle systems with first-order or second-order nonintegrable constraints are typical examples of this kind. Motion control of planar underactuated vehicle systems has received much consideration in the last two decades due to its intrinsic nonlinear properties and practical applications [1]–[3]. As a consequence of the underactuation, planar underactuated vehicles with zero gravitational and buoyant field do not meet the Brockett's necessary condition [4] and thus cannot be asymptotically stabilized by continuous pure-state feedback [5]. Therefore, in contrast with the case of fully-actuated systems, set-point stabilization cannot be considered as a special case of trajectory tracking.

Controlled collective behaviors of multi-vehicle systems are of particular interest in recent years due to their potential applications ranging from industry to military [6]. The distributed formation control problem, which can be considered as classical stabilization or trajectory tracking control problem extended to the multi-agent systems, is one of the most actively studied topics within the field of control engineering. The distributed formation control consists of making all the agents form a predefined geometrical configuration through *local* interactions with or without a group reference [7]. In other words, each agent uses only *local* information/measurements to achieve a *global* formation task. Among various control schemes, the leader-follower strategy is of particular significance in many applications due to its simplicity and scalability [8]. Within this framework, many research articles have addressed the formation tracking control problems for mobile robots [9], underactuated surface vessels [10], and aircraft [11].

Compared with formation tracking control, the formation stabilization control problem is more difficult and fewer work has been performed on this subject. In [12], a formation stabilization control law was proposed for multiple underactuated surface vessels but using the global position measurements. In [13], a fixed-time controller was proposed for a class of nonholonomic systems to address the formation stabilization problem. However, each vessel is required to know the information of its neighbor's neighbor. In [14], a distributed control law was proposed for unicycles to solve the rendezvous control problem. A consensus-based controller was developed to address the formation stabilization problem for a network of nonholonomic mobile robots in [15]. In [16], a uniform δ-persistently exciting (uδ-PE) controller was proposed for mobile robot networks and uniform global asymptotic stability (UGAS) for the origin of the closed-loop system was first established in the literature. It is noted that while there are several approaches to design controllers for different kinds of planar underactuated vehicles, they are heavily dependent on the particular structures of the vehicles. In practical applications, the vehicles may be of different types. Thus, it is more practical if the controller can be applied to various forms of planar vehicles.

In this paper, we develop a smooth time-varying leader-follower formation stabilization control framework for a class of planar underactuated vehicle networks. We do not assume any particular structure of the internal dynamics of each vehicle but rather use a *generic* Euler-Lagrangian (EL) model. The proposed control law requires only neighbor-to-neighbor information exchange, and does not require any global position measurements. The control design is developed based on passivity, partial stability theory and uδ-PE, and guarantees global asymptotic stability (GAS) for the origin of the closed-loop system. Furthermore, the structure of the controller is simple compared to the existing controllers in the literature.

II. Preliminaries and Problem Formulation

A. Notations

Let $\mathbb{R}^n$ represent the n-dimensional Euclidean space; $\mathbb{R}_{\geq 0}$ the set of all non-negative real numbers; $|\cdot|$ the Euclidean norm of vectors in $\mathbb{R}^n$. For any constant $\rho > 0$, we use the notations $\mathscr{B}_\rho := \{x \in \mathbb{R}^n : |x| < \rho\}$ and $\bar{\mathscr{B}}_\rho := \{x \in \mathbb{R}^n : |x| \leq$

This research was supported in part by the U.S. Office of Naval Research under Grant N00014-19-1-2255. (Corresponding author: Sergey Nersesov)

Bo Wang, Hashem Ashrafiuon, and Sergey Nersesov are with the Department of Mechanical Engineering, Villanova University, Villanova, PA, USA. Email: {bwang6, hashem.ashrafiuon, sergey.nersesov}@villanova.edu

$\rho\}$ to denote open and closed ball of radius ρ, respectively. For a full-rank matrix $G \in \mathbb{R}^{n\times m}$ with $m < n$, we denote the generalized inverse as $G^\dagger := [G^\top G]^{-1}G^\top$, and define $\mathtt{sym}(G) := \frac{1}{2}(G+G^\top)$. Throughout this paper, we omit the arguments of functions when they are clear from the context. For multi-agent systems, we use the bold and non-italicized subscript $\mathbf{i}$ to denote the index of an agent.

B. Model of Planar Underactuated Vehicles

Without loss of generality, a planar underactuated vehicle can be modeled as a 3-DOF planar rigid body with only two independent control inputs. The motion of a single vehicle $\mathbf{i}$ in the network is described by assigning a body-fixed reference frame $\{x_{b\mathbf{i}}y_{b\mathbf{i}}\}$ to its center of mass located at $(x_\mathbf{i}, y_\mathbf{i})$ and its orientation angle $\theta_\mathbf{i}$ with respect to a fixed inertial reference frame $\{XY\}$, as shown in Fig 1. The mathematical model of the planar underactuated vehicle $\mathbf{i}$ can be written in the EL form [3], [17]

$$\dot{q}_\mathbf{i} = J(q_\mathbf{i})v_\mathbf{i}, \tag{1a}$$

$$M\dot{v}_\mathbf{i} + C(v_\mathbf{i})v_\mathbf{i} + D(v_\mathbf{i})v_\mathbf{i} = G\tau_\mathbf{i}, \tag{1b}$$

where $q_\mathbf{i} = [x_\mathbf{i}, y_\mathbf{i}, \theta_\mathbf{i}]^\top$ is the configuration of the $\mathbf{i}$th vehicle; $v_\mathbf{i} = [v_{x\mathbf{i}}, v_{y\mathbf{i}}, \omega_\mathbf{i}]^\top$ is the generalized velocity vector consisting of the velocity of the center of mass $(v_{x\mathbf{i}}, v_{y\mathbf{i}})$ in the body-fixed frame $\{x_{b\mathbf{i}}y_{b\mathbf{i}}\}$ and its angular velocity $\omega_\mathbf{i}$; $\tau_\mathbf{i} = [\tau_{1\mathbf{i}}, \tau_{2\mathbf{i}}]^\top$ is the control input vector; $J(q_\mathbf{i})$ is the orthogonal kinematic transformation matrix given by

$$J(q_\mathbf{i}) = \begin{bmatrix} \cos(\theta_\mathbf{i}) & -\sin(\theta_\mathbf{i}) & 0 \\ \sin(\theta_\mathbf{i}) & \cos(\theta_\mathbf{i}) & 0 \\ 0 & 0 & 1 \end{bmatrix}; \tag{2}$$

M is the inertia matrix; $C(v_\mathbf{i})$ is the Coriolis and centrifugal matrix; $D(v_\mathbf{i})$ is the damping matrix; and G is the input matrix. All matrices above are assumed to be in appropriate dimensions. Three well-known properties associated with the EL system (1a), (1b) are as follows.

Property 1: For a single rigid body, the inertia matrix M is constant, symmetric and positive definite, and the Coriolis and centrifugal matrix $C(v_\mathbf{i})$ is skew-symmetric.

Property 2: The damping matrix $D(v_\mathbf{i})$ is symmetric and positive semi-definite.

Property 3: For the system (1a), (1b), the differential equation $M\dot{s}_\mathbf{i} + C(v_\mathbf{i})s_\mathbf{i} + D(v_\mathbf{i})s_\mathbf{i} = G\tau_\mathbf{i}$ defines an input–output mapping $\tau_\mathbf{i} \mapsto y_\mathbf{i} := G^\top s_\mathbf{i}$, which is passive with the storage function $E_K := \frac{1}{2}s_\mathbf{i}^\top M s_\mathbf{i}$. Furthermore, if $D(\cdot)$ is positive definite, then the mapping $\tau_\mathbf{i} \mapsto y_\mathbf{i}$ is output strictly passive.

We make the following assumption.

Assumption 1: (i.) For each vehicle $\mathbf{i}$, assume that the inertia matrix M is diagonal, i.e., $M = \mathrm{diag}(m_{11}, m_{22}, m_{33})$. (ii.) Assume that the surge force and the yaw torque are two independent control inputs. That is, the input matrix G may be written as

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \tag{3}$$

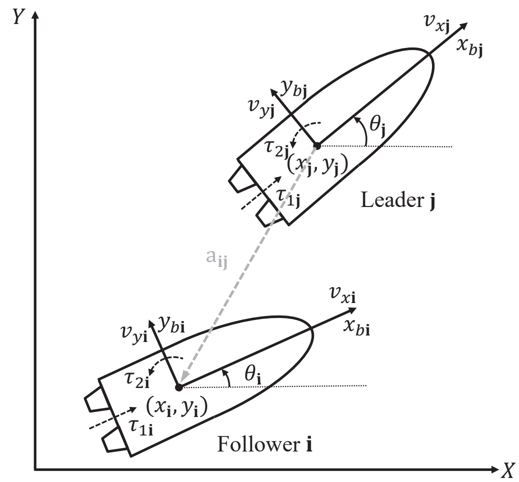


Fig. 1. Top view of the leader-follower formation of planar underactuated vehicles **i** and **j**.

which implies that the underactuation is in the sway direction, i.e., $v_{y\mathbf{i}}$-equation. (iii.) Assume that for each vehicle, the damping force in the sway direction satisfies $[D(v_\mathbf{i})]_{(2,2)} > 0$ for all $v_{y\mathbf{i}} \neq 0$, and $v_{y\mathbf{i}}/[D(v_\mathbf{i})]_{(2,2)} \to 0$ as $v_{y\mathbf{i}} \to 0$, where $[D(v_\mathbf{i})]_{(2,2)}$ denotes the $(2,2)$-element of $D(v_\mathbf{i})$.

C. Graph Theory

We use graph theory to define the communication interaction among the vehicles. Consider a network of $N+1$ planar underactuated vehicles, where the vehicles are numbered $\mathbf{i} = 0, 1, \ldots, N$ with $\mathbf{0}$ representing the group leader and $\mathbf{1}, \ldots, \mathbf{N}$ the follower agents. The network topology of the vehicles is defined by a directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ where $\mathscr{V} = \{\mathbf{0}, \mathbf{1}, \ldots, \mathbf{N}\}$ and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ represent its sets of vertices and edges, respectively. The set of neighboring nodes with edges connected to node $\mathbf{i}$ is denoted by $\mathscr{N}_\mathbf{i} = \{\mathbf{j} \in \mathscr{V} : (\mathbf{i}, \mathbf{j}) \in \mathscr{E}\}$. The edges represent communication between the nodes such that (follower) node $\mathbf{i}$ obtains information from (leader) node $\mathbf{j}$ for feedback control purposes, if $\mathbf{j} \in \mathscr{N}_\mathbf{i}$, as shown in Fig. 1. The constant weighted adjacency matrix $\mathscr{A} = [a_{\mathbf{ij}}]$ associated with $\mathscr{G}$ is defined in accordance with the rule that $a_{\mathbf{ij}} > 0$ in the case that $\mathbf{j} \in \mathscr{N}_\mathbf{i}$ and $a_{\mathbf{ij}} = 0$ otherwise. For the group leader, we have $a_{\mathbf{0j}} \equiv 0$ for all $\mathbf{j} \in \mathscr{V}$. We also assume that $a_{\mathbf{ii}} = 0$ for all $\mathbf{i} \in \mathscr{V}$. For more details on algebraic graph theory, see [6].

D. Problem Formulation

The geometric pattern of the vehicle network in terms of planar position may be defined by a set of constant offset vectors $\{d_{\mathbf{ij}} := (d^x_{\mathbf{ij}}, d^y_{\mathbf{ij}}) \in \mathbb{R}^2 : \mathbf{i}, \mathbf{j} \in \mathscr{V}, \mathbf{i} \neq \mathbf{j}\}$.

Formation Stabilization Control Problem: Assume that group leader is static, i.e., $(x_\mathbf{0}(t), y_\mathbf{0}(t), \theta_\mathbf{0}(t)) \equiv (x_\mathbf{0}, y_\mathbf{0}, \theta_\mathbf{0})$. Design a distributed controller for each follower agent such that it coordinates its motion relative to one or more of its neighbors, and the network asymptotically converges to a predefined geometric pattern with a desired orientation, i.e., design control laws for system (1a), (1b) such that

$$\lim_{t\to\infty}\begin{bmatrix} x_{\mathbf{i}}(t)-x_{\mathbf{j}}(t)-d^x_{\mathbf{ij}} \\ y_{\mathbf{i}}(t)-y_{\mathbf{j}}(t)-d^y_{\mathbf{ij}}\end{bmatrix}=0, \tag{4}$$

$$\lim_{t\to\infty}\left(\theta_{\mathbf{i}}(t)-\theta_{\mathbf{j}}(t)\right)=0. \tag{5}$$

E. Technical Lemmas

1) Partial stability conditions for UGAS of interconnected systems: For basic definitions and the use of partial stability in the analysis of interconnected systems, the readers are referred to [18], [19]. Consider the following time-varying interconnected system

$$\Sigma_1: \dot{x}_1 = f_1(t,x_1,x_2),\ x_1(t_0)=x_{10},\ t_0\geq 0, \tag{6}$$

$$\Sigma_2: \dot{x}_2 = f_2(t,x_1,x_2),\ x_2(t_0)=x_{20}, \tag{7}$$

where $x=(x_1,x_2)\in\mathbb{R}^{n_1}\times\mathbb{R}^{n_2}$. We assume that the functions f_1, f_2 are continuous in their arguments, locally Lipschitz in (x_1,x_2), uniformly in t, and the origin $(x_1,x_2)=(0,0)$ is an equilibrium point. For nonlinear time-varying system (6), (7), we give sufficient conditions to guarantee the partial stability of origin.

Theorem 1: Suppose that f_2 is continuously differentiable. Then, the origin of the interconnected system (6), (7) is UGAS if the following conditions hold.

1) (Partial stability with respect to x_1) There exist a continuously differentiable function $V_1:\mathbb{R}\times\mathbb{R}^{n_1}\times\mathbb{R}^{n_2}\to\mathbb{R}_{\geq 0}$, functions $\alpha_1,\alpha_2\in\mathscr{K}_\infty$, and a positive definite function $W_1:\mathbb{R}^{n_1}\to\mathbb{R}$ such that

$$\alpha_1(|x_1|)\leq V_1(t,x_1,x_2)\leq\alpha_2(|x_1|),$$
$$\dot{V}_1(t,x_1,x_2)\leq -W_1(x_1),$$

for all $(t,x_1,x_2)\in\mathbb{R}\times\mathbb{R}^{n_1}\times\mathbb{R}^{n_2}$.

2) (0-UGAS of Σ_2) There exist a continuously differentiable function $V_2:\mathbb{R}\times\mathbb{R}^{n_2}\to\mathbb{R}_{\geq 0}$, functions $\alpha_3,\alpha_4\in\mathscr{K}_\infty$, function $\alpha_5\in\mathscr{K}$, and a positive definite function $W_2:\mathbb{R}^{n_2}\to\mathbb{R}$ such that

$$\alpha_3(|x_2|)\leq V_2(t,x_2)\leq\alpha_4(|x_2|),$$
$$\frac{\partial V_2}{\partial t}+\frac{\partial V_2}{\partial x_2}f_2(t,0,x_2)\leq -W_2(x_2),$$
$$\left|\frac{\partial V_2}{\partial x_2}\right|\leq\alpha_5(|x_2|),$$

for all $(t,x_2)\in\mathbb{R}\times\mathbb{R}^{n_2}$.

3) ($|x_1|$ is small order of W_1) The function W_1 satisfies

$$\lim_{|x_1|\to\infty}\frac{|x_1|}{W_1(x_1)}=0. \tag{8}$$

Proof: Along the trajectories of (6), (7), we have

$$\dot{V}_2\leq -W_2(x_2)+\left|\frac{\partial V_2}{\partial x_2}\right|\left|\frac{\partial f_2}{\partial x_1}\right||x_1|.$$

Since V_2 is continuously differentiable and f_2 is continuous and Lipshitz, it follows that for each $r>0$ there exist $c_1>0$ and $c_2>0$ such that $|\partial V_2/\partial x_2|\leq c_1$ and $|\partial f_2/\partial x_1|\leq c_2$ for all $t\geq 0$, and for all $(x_1,x_2)\in\bar{\mathscr{B}}_r$. Then, consider a Lyapunov candidate $V=\kappa V_1+V_2$, where κ is a positive constant. Along the trajectories of (6), (7), we have

$$\dot{V}(t,x_1,x_2)\leq -\kappa W_1(x_1)\left[1-\frac{c_1c_2}{\kappa W_1(x_1)}|x_1|\right]-W_2(x_2). \tag{9}$$

It follows from (8), (9) that the system (6), (7) is uniformly globally bounded (UGB) by choosing κ sufficiently large. It follows from [19, Theorem 3.1] that the origin of system (6), (7) is uniformly asymptotically stable. Thus, there exists $\delta>0$ such that $|x(t_0)|<\delta\Rightarrow|x(t,t_0,x(t_0))|\to 0$ as $t\to\infty$. The uniform global attractivity follows from the fact that κ can be chosen arbitrarily large such that the trajectory of (6), (7) with initial conditions starting in $\bar{\mathscr{B}}_r$ enters the domain of attraction $\mathscr{B}_\delta$ for any $r>0$. ■

2) Matrosov's theorem: Our main result also relies on Matrosov's theorem concerning the differential equation $\dot{x}=f(t,x)$ with an equilibrium point at the origin.

Theorem 2 (Matrosov's theorem [20]): Suppose that there exist a continuous function $V^\star:\mathbb{R}^n\to\mathbb{R}_{\geq 0}$; continuously differentiable functions $V:\mathbb{R}_{\geq 0}\times\mathbb{R}^n\to\mathbb{R}$ and $W:\mathbb{R}_{\geq 0}\times\mathbb{R}^n\to\mathbb{R}$, functions; $\alpha_1,\alpha_2\in\mathscr{K}_\infty$, and for each $R>0$, there exists $L>0$ such that

(a) W and f satisfy

$$\max\{|W(t,x)|,|f(t,x)|\}\leq L,\quad\forall(t,x)\in\mathbb{R}_{\geq 0}\times\bar{\mathscr{B}}_R;$$

(b) V is positive definite decrescent and $\dot{V}$ is negative semi-definite, i.e., for all $(t,x)\in\mathbb{R}_{\geq 0}\times\mathbb{R}^n$

$$\alpha_1(|x|)\leq V(t,x)\leq\alpha_2(|x|),$$
$$\dot{V}(t,x)\leq -V^\star(x)\leq 0;$$

(c) the function $\dot{W}(t,x)$ is non-zero definite on

$$\mathscr{M}:=\left\{x\in\bar{\mathscr{B}}_R:V^\star(x)=0\right\}.$$

Then, the origin of $\dot{x}=f(t,x)$ is UGAS.

III. Formation Control Design

For the leader-follower formation problem, we usually consider the problem for the follower $\mathbf{i}$ as tracking a reference leader similar to [1], [3], [10], [16], [21]. The basic idea is to calculate the dynamics of the tracking error $(q_{\mathbf{i}}-\bar{q}_{\mathbf{i}},v_{\mathbf{i}}-\bar{v}_{\mathbf{i}})$, where

$$\bar{q}_{\mathbf{i}}(t):=\begin{bmatrix}\bar{x}_{\mathbf{i}}(t)\\ \bar{y}_{\mathbf{i}}(t)\\ \bar{\theta}_{\mathbf{i}}(t)\end{bmatrix}=\frac{1}{\sum_{\mathbf{j}\in\mathscr{N}_{\mathbf{i}}}a_{\mathbf{ij}}}\sum_{\mathbf{j}\in\mathscr{N}_{\mathbf{i}}}a_{\mathbf{ij}}\begin{bmatrix}x_{\mathbf{j}}(t)+d^x_{\mathbf{ij}}\\ y_{\mathbf{j}}(t)+d^y_{\mathbf{ij}}\\ \theta_{\mathbf{j}}(t)\end{bmatrix},$$

$$\bar{v}_{\mathbf{i}}(t):=\begin{bmatrix}\bar{v}_{x\mathbf{i}}(t)\\ \bar{v}_{y\mathbf{i}}(t)\\ \bar{\omega}_{\mathbf{i}}(t)\end{bmatrix}=\frac{1}{\sum_{\mathbf{j}\in\mathscr{N}_{\mathbf{i}}}a_{\mathbf{ij}}}\sum_{\mathbf{j}\in\mathscr{N}_{\mathbf{i}}}a_{\mathbf{ij}}\begin{bmatrix}v_{x\mathbf{j}}(t)\\ v_{y\mathbf{j}}(t)\\ \omega_{\mathbf{j}}(t)\end{bmatrix},$$

and try to stabilize this error system. However, the error system often becomes very complex. Thus, instead of using $\bar{v}_{\mathbf{i}}$, we define the new reference velocity in the body-fixed frame $\{x_{b\mathbf{i}}y_{b\mathbf{i}}\}$ as $\hat{v}_{\mathbf{i}}:=J(q_{\mathbf{i}})^\top\dot{\bar{q}}_{\mathbf{i}}$. Correspondingly, for agent $\mathbf{i}$, the error vectors in the body-fixed frame $\{x_{b\mathbf{i}}y_{b\mathbf{i}}\}$ are defined as $\tilde{q}^b_{\mathbf{i}}=[\tilde{x}^b_{\mathbf{i}},\tilde{y}^b_{\mathbf{i}},\tilde{\theta}_{\mathbf{i}}]^\top:=J(q_{\mathbf{i}})^\top(q_{\mathbf{i}}-\bar{q}_{\mathbf{i}})$, and $\tilde{v}_{\mathbf{i}}=[\tilde{v}_{x\mathbf{i}},\tilde{v}_{y\mathbf{i}},\tilde{\omega}_{\mathbf{i}}]^\top:=(v_{\mathbf{i}}-\hat{v}_{\mathbf{i}})$. Clearly, since $J(q_{\mathbf{i}})$ is invertible, stabilization of $\tilde{q}^b_{\mathbf{i}}$, $\tilde{v}_{\mathbf{i}}$ implies that $q_{\mathbf{i}}(t)\to\bar{q}_{\mathbf{i}}(t)$ and $\dot{q}_{\mathbf{i}}(t)\to\dot{\bar{q}}_{\mathbf{i}}(t)$ as $t\to\infty$ which solves formation control problem (5), (6). Let us consider the following modified PD+ controller

$$\tau_{\mathbf{i}} = G^{\dagger}\left[M\dot{\hat{v}}_{\mathbf{i}} + C(v_{\mathbf{i}})\hat{v}_{\mathbf{i}} + D(v_{\mathbf{i}})\hat{v}_{\mathbf{i}} - K_{p\mathbf{i}}\tilde{q}_{\mathbf{i}}^{b} - K_{d\mathbf{i}}\tilde{v}_{\mathbf{i}} + u_{\mathbf{i}}\right], \tag{10}$$

where $K_{p\mathbf{i}} > 0$ and $K_{d\mathbf{i}} > 0$ are constant, diagonal control gain matrices; $u_{\mathbf{i}}$ is a new control input which will be designed later. We have the following result.

Proposition 1: Consider the planar underactuated vehicle (1a), (1b) satisfying Assumption 1. Then, under the modified PD+ control law (10) with $u_{\mathbf{i}} \equiv 0$, the origin for the $(\tilde{x}_{\mathbf{i}}^{b}, \tilde{\theta}_{\mathbf{i}}, \tilde{v}_{x\mathbf{i}}, \tilde{v}_{y\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$-subsystem is UGAS, and the solutions of the closed-loop system are UGB.

Proof: Consider the function

$$V_{\mathbf{i}}(\tilde{q}_{\mathbf{i}}, \tilde{v}_{\mathbf{i}}) = \frac{1}{2}\left[\tilde{v}_{\mathbf{i}}^{\top}\left(GG^{\dagger}\right)M\tilde{v}_{\mathbf{i}} + (\tilde{q}_{\mathbf{i}}^{b})^{\top}\left(GG^{\dagger}\right)K_{p\mathbf{i}}\tilde{q}_{\mathbf{i}}^{b}\right],$$

which is positive definite with respect to the error vector $(\tilde{x}_{\mathbf{i}}^{b}, \tilde{\theta}_{\mathbf{i}}, \tilde{v}_{x\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$. Taking time derivative along the trajectories of the closed-loop system, we have

$$\dot{V}_{\mathbf{i}} = -\tilde{v}_{\mathbf{i}}^{\top}\,\mathtt{sym}\left\{\left(GG^{\dagger}\right)\left[D(v_{\mathbf{i}}) + K_{d\mathbf{i}}\right]\right\}\tilde{v}_{\mathbf{i}} \le 0, \tag{11}$$

Consequently, we have $(GG^{\dagger})\tilde{v}_{\mathbf{i}} \in L_2$, and the origin for the $(\tilde{x}_{\mathbf{i}}^{b}, \tilde{\theta}_{\mathbf{i}}, \tilde{v}_{x\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$-subsystem is UGS. It also follows from LaSalle-Yoshizawa theorem that $(\tilde{v}_{x\mathbf{i}}, \tilde{\omega}_{\mathbf{i}}) \to 0$ as $t \to \infty$. If we consider $\tilde{v}_{y\mathbf{i}}(t)$ as a time-varying signal, then the origin of the $(\tilde{v}_{x\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$-subsystem is uniformly globally exponentially stable. Then, the $(\tilde{v}_{x\mathbf{i}}, \tilde{v}_{y\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$-subsystem is globally exponential stable with respect to $(\tilde{v}_{x\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$ uniformly in $\tilde{v}_{y\mathbf{i}}(0)$ (i.e., partial stability with respect to $(\tilde{v}_{x\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$). It also follows from the Assumption 1 item (iii.) that the origin of $\tilde{v}_{y\mathbf{i}}$-dynamics is UGAS when $(\tilde{v}_{x\mathbf{i}}, \tilde{\omega}_{\mathbf{i}}) \equiv (0,0)$ (i.e., 0-UGAS of $\tilde{v}_{y\mathbf{i}}$-subsystem). Therefore, we conclude that the $(\tilde{v}_{x\mathbf{i}}, \tilde{v}_{y\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$-subsystem is UGAS according to Theorem 1. Moreover, the condition $v_{y\mathbf{i}}/[D_{\mathbf{i}}(v_{\mathbf{i}})]_{(2,2)} \to 0$ as $v_{y\mathbf{i}} \to 0$ implies that $\tilde{v}_{y\mathbf{i}} \in L_1$ and $\tilde{y}_{\mathbf{i}}^{b} \in L_\infty$. Thus, we conclude that the solutions of the closed-loop system are UGB.

Next, for the $(\tilde{x}_{\mathbf{i}}^{b}, \tilde{\theta}_{\mathbf{i}}, \tilde{v}_{x\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$-subsystem, consider the auxiliary function $W_{\mathbf{i}} = (\tilde{q}_{\mathbf{i}}^{b})^{\top}(GG^{\dagger})M\tilde{v}_{\mathbf{i}}$. Taking time derivative of $W_{\mathbf{i}}$ along trajectories of the closed-loop system, we have

$$\dot{W}_{\mathbf{i}} = (\dot{\tilde{q}}_{\mathbf{i}}^{b})^{\top}(GG^{\dagger})M\tilde{v}_{\mathbf{i}} + (\tilde{q}_{\mathbf{i}}^{b})^{\top}(GG^{\dagger})M\dot{\tilde{v}}_{\mathbf{i}}.$$

Then, evaluating $\dot{W}_{\mathbf{i}}$ on the set $\mathscr{M} := \{\tilde{v}_{\mathbf{i}} = 0\}$ yields

$$\dot{W}_{\mathbf{i}}|_{\mathscr{M}} = -(\tilde{q}_{\mathbf{i}}^{b})^{\top}(GG^{\dagger})K_{p\mathbf{i}}\tilde{q}_{\mathbf{i}}^{b} \le 0.$$

Thus, $\dot{W}_{\mathbf{i}}$ is non-zero definite on the set $\mathscr{M}$. It follows from the Matrosov's Theorem 2 that the origin for the $(\tilde{x}_{\mathbf{i}}^{b}, \tilde{\theta}_{\mathbf{i}}, \tilde{v}_{x\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$-subsystem is UGAS. Therefore, we conclude that the origin of the $(\tilde{x}_{\mathbf{i}}^{b}, \tilde{\theta}_{\mathbf{i}}, \tilde{v}_{x\mathbf{i}}, \tilde{v}_{y\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$-subsystem is UGAS by considering $\tilde{y}_{\mathbf{i}}^{b}(t)$ as a bounded time-varying signal. ■

Under the modified passivity-based PD+ controller (10) with $u_{\mathbf{i}} \equiv 0$, the velocity error vector $\tilde{v}_{\mathbf{i}}(t) \to 0$, and the position error in the body-fixed frame $(\tilde{x}_{\mathbf{i}}^{b}(t), \tilde{\theta}_{\mathbf{i}}(t)) \to 0$ as $t \to \infty$. However, due to the underactuation, the position error $\tilde{y}_{\mathbf{i}}^{b}(t)$ may converge only to a constant which is not necessarily zero. Denote the position error in the global frame by $(\tilde{x}_{\mathbf{i}}, \tilde{y}_{\mathbf{i}}) := (x_{\mathbf{i}} - \bar{x}_{\mathbf{i}}, y_{\mathbf{i}} - \bar{y}_{\mathbf{i}})$. Although

$$\tilde{x}_{\mathbf{i}}^{b}(t) = \begin{bmatrix}\cos(\theta_{\mathbf{i}}) & \sin(\theta_{\mathbf{i}})\end{bmatrix}\begin{bmatrix}\tilde{x}_{\mathbf{i}}(t) & \tilde{y}_{\mathbf{i}}(t)\end{bmatrix}^{\top} \to 0 \tag{12}$$

does not imply that $(\tilde{x}_{\mathbf{i}}(t), \tilde{y}_{\mathbf{i}}(t)) \to 0$ due to the rank deficiency of $[\cos(\theta_{\mathbf{i}}), \sin(\theta_{\mathbf{i}})]$, a persistently exciting $\theta_{\mathbf{i}}(t)$ will guarantee that the position error $(\tilde{x}_{\mathbf{i}}(t), \tilde{y}_{\mathbf{i}}(t)) \to 0$ as $t \to \infty$.

Proposition 2: Assume that the velocity error vector $\tilde{v}_{\mathbf{i}}(t) \in L_1 \cap L_\infty$, and that $\omega_{\mathbf{i}}(t)$ is persistently exciting ($\omega_{\mathbf{i}} \in$ PE), that is, there exist constants $T_{\mathbf{i}}, \mu_{\mathbf{i}} > 0$ such that

$$\int_{t}^{t+T_{\mathbf{i}}} \omega_{\mathbf{i}}(\tau)^2 \mathrm{d}\tau \ge \mu_{\mathbf{i}}, \quad \forall t \ge 0. \tag{13}$$

Then, $\tilde{x}_{\mathbf{i}}^{b}(t) \to 0$ as $t \to \infty$ implies that $(\tilde{x}_{\mathbf{i}}(t), \tilde{y}_{\mathbf{i}}(t)) \to 0$ as $t \to \infty$.

Proof: Note that $J(q_{\mathbf{i}})$ is an orthogonal matrix, and $\tilde{v}_{\mathbf{i}}(t) \in L_1 \cap L_\infty$ implies that $(\dot{q}_{\mathbf{i}}(t) - \dot{\bar{q}}_{\mathbf{i}}(t)) \in L_1 \cap L_\infty$. Also, $\tilde{v}_{\mathbf{i}}(t) \to 0$ implies that $(\dot{q}_{\mathbf{i}}(t) - \dot{\bar{q}}_{\mathbf{i}}(t)) \to 0$ as $t \to \infty$. Thus, by integrating both sides, we conclude that $(\tilde{x}_{\mathbf{i}}(t), \tilde{y}_{\mathbf{i}}(t)) \to$ const. Now, consider the following equation

$$c_1\cos(\theta_{\mathbf{i}}) + c_2\sin(\theta_{\mathbf{i}}) = 0, \tag{14}$$

where c_1, c_2 are constants. If one of c_1 and c_2 is non-zero, then the equation (14) has only isolate solutions $\theta_{\mathbf{i}} =$ const. On the other hand, by the filter property of persistently exciting signals, $\omega_{\mathbf{i}} \in$ PE implies that $\theta_{\mathbf{i}}$ does not converge to a constant as $t \to \infty$. Thus, by contradiction and the continuity of (12), we conclude that $(\tilde{x}_{\mathbf{i}}(t), \tilde{y}_{\mathbf{i}}(t)) \to 0$ as $t \to \infty$. ■

It follows from Propositions 1 and 2 that if the angular velocity of the vehicle **i** is PE, then the modified PD+ controller (10) with $u_{\mathbf{i}} \equiv 0$ may be used to solve the formation problem. However, in the cases of formation stabilization, the angular velocity of the vehicle **i** converges to zero and thus the PE property is lost. In this case, we will use $u_{\mathbf{i}}$ as a "PE perturbation" on the angular motion to prevent $(\tilde{x}_{\mathbf{i}}(t), \tilde{y}_{\mathbf{i}}(t))$ converging to a non-zero constant. The new control input $u_{\mathbf{i}}$ is defined as

$$u_{\mathbf{i}} = \begin{bmatrix}0 & 0 & \alpha_{\mathbf{i}}(t, \tilde{y}_{\mathbf{i}}^{b})\end{bmatrix}^{\top}, \tag{15}$$

where $\alpha_{\mathbf{i}}(t, \tilde{y}_{\mathbf{i}}^{b}) = k_{\rho\mathbf{i}}\rho_{\mathbf{i}}(t)\tilde{y}_{\mathbf{i}}^{b}(t)$, $k_{\rho\mathbf{i}} > 0$ is a constant, and the time-varying signal $\rho_{\mathbf{i}}(t)$ is PE, continuously differentiable, and bounded with bounded first derivative. Note that the excitation property of $\alpha_{\mathbf{i}}$ is reminiscent of $u\delta$-PE with respect to $\tilde{y}_{\mathbf{i}}^{b}$ [22], i.e., for each $\delta > 0$ there exist $T, \mu > 0$ such that

$$\left|\tilde{y}_{\mathbf{i}}^{b}(t)\right| > \delta \Rightarrow \int_{t}^{t+T} \alpha_{\mathbf{i}}(\tau, \tilde{y}_{\mathbf{i}}^{b})^2 \mathrm{d}\tau > \mu, \quad \forall t \ge 0. \tag{16}$$

Proposition 3: Consider the planar underactuated vehicle (1a), (1b) satisfying Assumption 1. Then, under the modified PD+ control law (10) and (15), the origin for the $(\tilde{x}_{\mathbf{i}}^{b}, \tilde{y}_{\mathbf{i}}^{b}, \tilde{\theta}_{\mathbf{i}}, \tilde{v}_{x\mathbf{i}}, \tilde{v}_{y\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$-dynamics is GAS.

Proof: It follows from Proposition 1 that if $\alpha_{\mathbf{i}}(t, \tilde{y}_{\mathbf{i}}^{b}) \equiv 0$, the $(\tilde{x}_{\mathbf{i}}^{b}, \tilde{\theta}_{\mathbf{i}}, \tilde{v}_{x\mathbf{i}}, \tilde{v}_{y\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$-subsystem is UGAS to its origin. Furthermore, due to the damping term $K_{d\mathbf{i}}$ in the PD+ control law (10), the angular motion dynamics is input-to-state stable (ISS) by considering $\alpha_{\mathbf{i}}(t, \tilde{y}_{\mathbf{i}}^{b})$ as an input. It also follows from the proof in Proposition 2 that $\tilde{y}_{\mathbf{i}}^{b}(t)$ converges to a constant as $t \to \infty$. Now, assume that $\tilde{y}_{\mathbf{i}}^b(t)$ converges to a non-zero constant. Then, (16) implies $\alpha_{\mathbf{i}} \in$ PE, and from the filter property we have $\omega_{\mathbf{i}}(t) \in$ PE. Then, it follows from Proposition 2 that $(\tilde{x}_{\mathbf{i}}(t), \tilde{y}_{\mathbf{i}}(t)) \to 0$ as $t \to \infty$, which contradicts the assumption that $\tilde{y}_{\mathbf{i}}^b(t)$ converges to a non-zero constant. Thus, we conclude that $\tilde{y}_{\mathbf{i}}^b(t) \to 0$ as $t \to \infty$ by contradiction. The GAS of the origin comes from the ISS property when $\alpha_{\mathbf{i}}(t, \tilde{y}_{\mathbf{i}}^b) \to 0$ as $t \to \infty$. ■

Our main result comes from the previous rationale.

Theorem 3: Consider a network of heterogeneous planar underactuated vehicles satisfying Assumption 1. Then, the formation is achieved under the modified PD+ control law (10) and (15) if the directed communication graph contains a spanning tree.

Proof: By the assumption of the spanning tree topology in the communication graph and using Proposition 3, an immediate consequence of the claim is that for each vehicle $\mathbf{i}$ in the group, the origin for the $(\tilde{x}_{\mathbf{i}}^b, \tilde{y}_{\mathbf{i}}^b, \tilde{\theta}_{\mathbf{i}}, \tilde{v}_{x\mathbf{i}}, \tilde{v}_{y\mathbf{i}}, \tilde{\omega}_{\mathbf{i}})$-dynamics is GAS. It follows from the converse Lyapunov theorem that there exist a continuously differentiable function $\mathbf{V_i}: \mathbb{R} \times \mathbb{R}^6 \to \mathbb{R}_{\geq 0}$, $\varphi_{1\mathbf{i}}, \varphi_{2\mathbf{i}} \in \mathscr{K}_\infty$, and a positive definite function $\mathbf{W_i}$ such that $\varphi_{1\mathbf{i}}\left(\left|(\tilde{q}_{\mathbf{i}}^b, \tilde{v}_{\mathbf{i}})\right|\right) \leq \mathbf{V_i}\left(t, \tilde{q}_{\mathbf{i}}^b, \tilde{v}_{\mathbf{i}}\right) \leq \varphi_{2\mathbf{i}}\left(\left|(\tilde{q}_{\mathbf{i}}^b, \tilde{v}_{\mathbf{i}})\right|\right)$ and $\dot{\mathbf{V}}_{\mathbf{i}} \leq -\mathbf{W_i}\left((\tilde{q}_{\mathbf{i}}^b, \tilde{v}_{\mathbf{i}})\right)$. Then, define the Lyapunov candidate $\mathbf{V} := \sum_{\mathbf{i} \in \mathscr{V}} \sum_{\mathbf{j} \in \mathscr{N}_{\mathbf{i}}} a_{\mathbf{ij}} \mathbf{V_i}$. Note that if the communication graph contains a spanning tree, then the Lyapunov candidate $\mathbf{V}$ covers all the agents in the network. Taking the time derivative along the trajectories of the closed-loop system, we have that $\dot{\mathbf{V}} \leq -\sum_{\mathbf{i} \in \mathscr{V}} \sum_{\mathbf{j} \in \mathscr{N}_{\mathbf{i}}} a_{\mathbf{ij}} \mathbf{W_i}\left((\tilde{q}_{\mathbf{i}}^b, \tilde{v}_{\mathbf{i}})\right)$. Thus, the formation error converges to zero as $t \to \infty$, and we conclude that the formation is achieved if the communication graph contains a spanning tree. ■

IV. Applications and Numerical Simulations

A. Applications

Underactuated Surface Vessels. The EL equations for an underactuated surface vessel model with nonlinear hydrodynamic damping are given by (1a), (1b) with

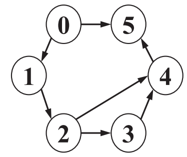
$$M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix}, C(v) = \begin{bmatrix} 0 & 0 & -m_{22}v_y \\ 0 & 0 & m_{11}v_x \\ m_{22}v_y & -m_{11}v_x & 0 \end{bmatrix},$$

$$D(v) = \begin{bmatrix} d_{11}|v_x|^{\alpha_{11}} & 0 & 0 \\ 0 & d_{22}|v_y|^{\alpha_{22}} & 0 \\ 0 & 0 & d_{33}|\omega|^{\alpha_{33}} \end{bmatrix}, G = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

where $m_{ii} > 0$; $d_{ii} > 0$ and $0 \leq \alpha_{ii} < 1$ for $i = 1,2,3$ [3], [23]. This model is also applicable to linear hydrodynamic damping with $\alpha_{ii} = 0$, which is the model used in [2], [24]–[26]. The conditions in Assumption 1 can be verified directly and are satisfied for this model.

Wheeled Mobile Robots. Due to the nonholonomic constraints, the dimensions of the tangent (velocity) space is reduced. The EL equations for a nonholonomic mobile robot model are given by (1a), (1b) with

$$M = \begin{bmatrix} \tilde{m} & 0 \\ 0 & \tilde{I} \end{bmatrix}, C(v) = \begin{bmatrix} 0 & -md\omega \\ md\omega & 0 \end{bmatrix}, G = \begin{bmatrix} \frac{1}{r} & 0 \\ 0 & \frac{a}{r} \end{bmatrix},$$



$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Fig. 2. Directed communication topology and the weighted adjacency matrix used in the simulations.

and $D(v) = 0$, where $\tilde{m} = m + 2J/r^2, \tilde{I} = I + md^2 + a^2J/r^2$, and $m, d, I, J, a, r > 0$ are constants [9], [16], [27]. Although there is no v_y-dynamics in the model and the damping matrix $D(v)$ is zero, the nonholonomic constraint $v_y = d\omega$ suggests that the damping term introduced by the control law $K_{d\mathbf{i}}$ makes the dynamic equations output strictly passive. Thus, the modified PD+ control law (10), (15) can be applied to this model directly, and the GAS for the v_y-subsystem comes directly from the linear relationship between the ω-dynamics and the v_y-dynamics.

B. Numerical Simulations

Let us consider a group of six planar underactuated vehicles with the indices $\mathbf{0}-\mathbf{5}$. Agent $\mathbf{0}$ is the leader and agents $\mathbf{1}-\mathbf{5}$ are the followers with the communication topology graph and the weighted adjacency matrix as shown in Fig. 2. We assume that agents are underactuated surface vessels with linear hydrodynamic damping whose parameters are given as $m_{11} = 1.412, m_{22} = 1.982, m_{33} = 0.354, d_{11} = 3.436, d_{22} = 12.99, d_{33} = 0.864$. All the parameters are given in SI units. The desired geometric pattern in formation is assumed to be a regular hexagon with the side length $h = 2$, i.e., $(d_{\mathbf{10}}^x, d_{\mathbf{10}}^y) = (-1, -\sqrt{3})$, $(d_{\mathbf{21}}^x, d_{\mathbf{21}}^y) = (1, -\sqrt{3})$, $(d_{\mathbf{32}}^x, d_{\mathbf{32}}^y) = (0, 2)$, $(d_{\mathbf{43}}^x, d_{\mathbf{43}}^y) = (1, \sqrt{3})$, $(d_{\mathbf{54}}^x, d_{\mathbf{54}}^y) = (-1, \sqrt{3})$. The vehicles are assumed to be initially stationary at the coordinates $q_{\mathbf{0}}(0) = (0,0,0), q_{\mathbf{1}}(0) = (-5,-5,0), q_{\mathbf{2}}(0) = (-2,-6,1), q_{\mathbf{3}}(0) = (3,-5,1), q_{\mathbf{4}}(0) = (5,-5,1), q_{\mathbf{5}}(0) = (5,2,0)$.

We assume that the configuration for the group leader $\mathbf{0}$ is at the origin for all times $t \geq 0$. The control parameters are selected as $K_{p\mathbf{i}} = \text{diag}\{5,5,5\}$, $K_{d\mathbf{i}} = \text{diag}\{4,4,4\}$, $k_{\rho\mathbf{i}} = 2$ and $\rho_{\mathbf{i}}(t) = \sin(2t)$ for all $\mathbf{i} \in \mathscr{V}$.

The simulation results are shown in Figs. 3-4, where the root mean square (RMS) error shown in Fig. 3 is of the form of $\text{RMS}([\cdot]_{\mathbf{i}}) = (\frac{1}{n}\sum_{i=1}^n [\cdot]_{\mathbf{i}}^2)^{1/2}$. It can be seen from the figures that the formation errors approach zero after 40 seconds. As shown in Figs. 3-4, firstly, each vehicle converges to a small neighborhood of the desired formation position very fast. Then, it converges to the desired formation position with oscillation, and this convergence phase is slow. This oscillation is due to the $u\delta$-PE term $\alpha_{\mathbf{i}}$ introduced in the control law, and it is a common phenomenon in stabilization of nonholonomic and underactuated systems via smooth time-varying feedbacks.

V. Concluding Remarks

In this work, we presented a distributed control framework to address the formation stabilization control problem for

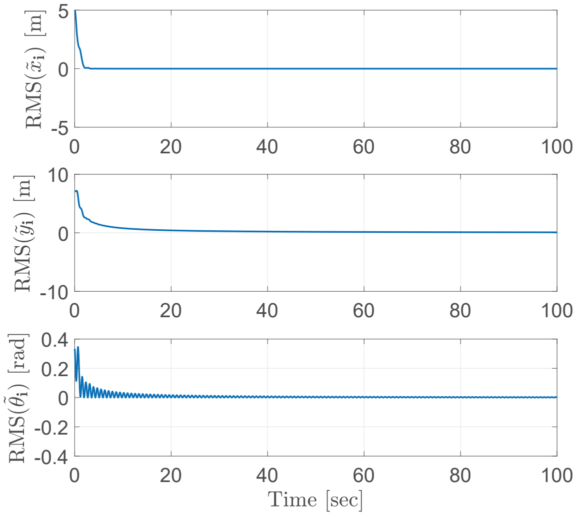


Fig. 3. Position paths in the $\{XY\}$ frame of the formation tracking.

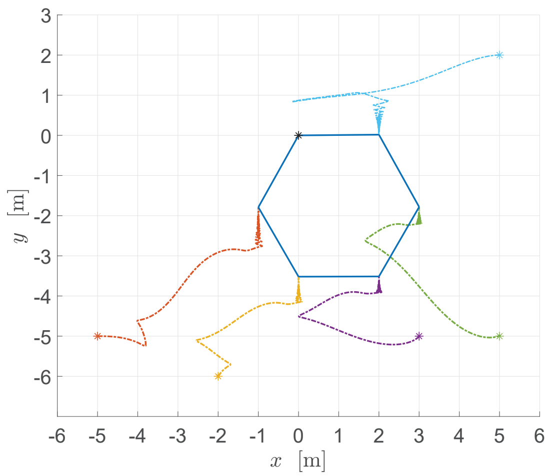


Fig. 4. Position paths in the $\{XY\}$ frame of the formation stabilization.

generic planar underactuated vehicle networks without global position measurements. The vehicles in the network are modeled as generic EL systems and we do not assume any particular structure of the internal dynamics of each vehicle. The control design is developed based on partial stability theory, Matrosov's theorem, and $u\delta$-PE concept, and guarantees GAS for the origin of the closed-loop system. The proposed controller has a PD+ form and is simple compared to existing controllers in the literature, and thus, it is practical and easy to implement.

References

[1] E. Panteley, E. Lefeber, A. Loria, and H. Nijmeijer, "Exponential tracking control of a mobile car using a cascaded approach," *IFAC Proceedings Volumes*, vol. 31, no. 27, pp. 201–206, 1998.

[2] Z.-P. Jiang, "Global tracking control of underactuated ships by Lyapunov's direct method," *Automatica*, vol. 38, no. 2, pp. 301–309, 2002.

[3] H. Ashrafiuon, S. Nersesov, and G. Clayton, "Trajectory tracking control of planar underactuated vehicles," *IEEE Trans. Automat. Contr.*, vol. 62, no. 4, pp. 1959–1965, 2016.

[4] R. W. Brockett, "Asymptotic stability and feedback stabilization," in *Differential geometric control theory*, 1983, pp. 181–191.

[5] C. I. Byrnes and A. Isidori, "On the attitude stabilization of rigid spacecraft," *Automatica*, vol. 27, no. 1, pp. 87–95, 1991.

[6] W. Ren and R. W. Beard, *Distributed consensus in multi-vehicle cooperative control*. London, U.K.: Springer-Verlag, 2008.

[7] W. Ren and Y. Cao, *Distributed coordination of multi-agent networks: Emergent problems, models, and issues*. London, U.K.: Springer-Verlag, 2011.

[8] L. Consolini, F. Morbidi, D. Prattichizzo, and M. Tosques, "Leader–follower formation control of nonholonomic mobile robots with input constraints," *Automatica*, vol. 44, no. 5, pp. 1343–1349, 2008.

[9] M. Ghasemi, S. G. Nersesov, G. Clayton, and H. Ashrafiuon, "Sliding mode coordination control for multiagent systems with underactuated agent dynamics," *Int. J. Control*, vol. 87, no. 12, pp. 2615–2633, 2014.

[10] B. Wang, S. Nersesov, and H. Ashrafiuon, "Formation control for underactuated surface vessel networks," in *ASME 2020 Dynamic Systems and Control Conference*. ASME, 2021, pp. DSCC2020–3178.

[11] F. Liao, R. Teo, J. L. Wang, X. Dong, F. Lin, and K. Peng, "Distributed formation and reconfiguration control of vtol uavs," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 1, pp. 270–277, 2016.

[12] W. Dong and J. A. Farrell, "Formation control of multiple underactuated surface vessels," *IET Control Theory Appl.*, vol. 2, no. 12, pp. 1077–1085, 2008.

[13] M. Defoort, G. Demesure, Z. Zuo, A. Polyakov, and M. Djemai, "Fixed-time stabilisation and consensus of non-holonomic systems," *IET Control Theory Appl.*, vol. 10, no. 18, pp. 2497–2505, 2016.

[14] A. Roza, M. Maggiore, and L. Scardovi, "A smooth distributed feedback for global rendezvous of unicycles," *IEEE Trans. Control Netw. Syst.*, vol. 5, no. 1, pp. 640–652, 2016.

[15] A. Bautista-Castillo, C. López-Franco, and E. Nuño, "Consensus-based formation control for multiple nonholonomic robots," in *2016 IEEE International Autumn Meeting on Power, Electronics and Computing (ROPEC)*. IEEE, 2016, pp. 1–6.

[16] M. A. Maghenem, A. Loria, and E. Panteley, "Cascades-based leader-follower formation-tracking and stabilization of multiple nonholonomic vehicles," *IEEE Trans. Automat. Contr.*, 2019.

[17] R. Ortega, J. A. L. Perez, P. J. Nicklasson, and H. J. Sira-Ramirez, *Passivity-based control of Euler-Lagrange systems: mechanical, electrical and electromechanical applications*. London, U.K.: Springer-Verlag, 1998.

[18] W. M. Haddad and V. Chellaboina, *Nonlinear dynamical systems and control: a Lyapunov-based approach*. Princeton, NJ, USA: Princeton university press, 2011.

[19] B. Wang, H. Ashrafiuon, and S. G. Nersesov, "The use of partial stability in the analysis of interconnected systems," *J. Dyn. Syst. Meas. Contr.*, vol. 143, p. 044501, 2021.

[20] N. Rouche, P. Habets, and M. Laloy, *Stability theory by Liapunov's direct method*. New York, NY, USA: Springer-Verlag, 1977.

[21] B. Wang, S. Nersesov, and H. Ashrafiuon, "A unified approach to stabilization, trajectory tracking, and formation control of planar underactuated vehicles," in *Proc. American Control Conf.* Denver, CO, USA: IEEE, 2020, pp. 5269–5274.

[22] A. Loría, E. Panteley, D. Popovic, and A. R. Teel, "A nested Matrosov theorem and persistency of excitation for uniform convergence in stable nonautonomous systems," *IEEE Trans. Automat. Contr.*, vol. 50, no. 2, pp. 183–198, 2005.

[23] X. Jin, "Fault tolerant finite-time leader–follower formation control for autonomous surface vessels with LOS range and angle constraints," *Automatica*, vol. 68, pp. 228–236, 2016.

[24] A. Behal, D. M. Dawson, W. E. Dixon, and Y. Fang, "Tracking and regulation control of an underactuated surface vessel with nonintegrable dynamics," *IEEE Trans. Automat. Contr.*, vol. 47, no. 3, pp. 495–500, 2002.

[25] K. D. Do, Z.-P. Jiang, and J. Pan, "Universal controllers for stabilization and tracking of underactuated ships," *Syst. Control Lett.*, vol. 47, no. 4, pp. 299–317, 2002.

[26] K. D. Do, Z.-P. Jiang, J. Pan, and H. Nijmeijer, "A global output-feedback controller for stabilization and tracking of underactuated ODIN: A spherical underwater vehicle," *Automatica*, vol. 40, no. 1, pp. 117–124, 2004.

[27] Y. Wang, Z. Miao, H. Zhong, and Q. Pan, "Simultaneous stabilization and tracking of nonholonomic mobile robots: A Lyapunov-based approach," *IEEE Trans. Contr. Syst. Technol.*, vol. 23, no. 4, pp. 1440–1450, 2015.