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Formation Regulation and Tracking Control for Nonholonomic Mobile Robot Networks Using Polar Coordinates

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***Abstract*— In this work, we solve the distributed leader-follower *simultaneous* formation regulation and tracking control problem for nonholonomic mobile robot networks without global position measurements. The controller is designed based on the formation error dynamics in polar coordinates. Under the assumption of the network topology containing a directed spanning tree, we establish global asymptotic stability and local exponential stability for the origin of the error system in polar coordinates using Lyapunov's direct method, and correspondingly, establish the exponential attractivity for the origin in Cartesian coordinates with the domain of attraction in any compact set. The proposed controller is time-invariant, continuous along trajectories, and does not require global position measurements. Simulations are presented to illustrate the robust performance of the proposed control method.**

I. INTRODUCTION

A. Motivation and Related works

The study of motion control of nonholonomic vehicles has been carried out by the control community since 1990's due to its intrinsic nonlinear properties and broad practical applications. One of the crucial characteristics of nonholonomic systems is that they cannot be asymptotically stabilized using continuous time-invariant state-feedback control law. This is because nonholonomic systems do not meet the Brockett's necessary condition for asymptotic stabilization [1]. To circumvent this obstacle, continuous time-varying [2], discontinuous time-invariant [3], and hybrid [4] feedback control systems have been developed in the literature. Moreover, instead of seeking the *asymptotic stability* of the closed-loop system, continuous time-invariant feedback also has been considered in the literature that guarantees only *attractivity* for the origin of the closed-loop system [5], [6]. Another key feature of nonholonomic systems is that there exists no *universal* continuous controller (even time-varying) that can track an arbitrary feasible trajectory [7]. Unlike the case of holonomic systems, the set-point regulation cannot be viewed as a special case of trajectory tracking control for nonholonomic systems. Thus, the set-point regulation problem and the trajectory tracking control problem are usually studied as two distinct problems in the literature.

Cooperative control of multi-vehicle systems has received much attention during the last decade due to advantages over single vehicles including higher efficiency, robustness, and flexibility in various missions [8]. Distributed formation control, which is to drive a group of agents to form a certain geometric pattern using *local* interactions, can be viewed as the classical stabilization or trajectory tracking control problem generalized to the multi-agent systems. Specifically, each follower in the network utilizes only *local* information to achieve the *global* formation task. Similar to the case of single nonholonomic systems, the formation stabilization and the formation tracking control problems for multi-nonholonomic-vehicle systems are typically studied as two separate problems. Thus, each agent needs to know the control task beforehand, and switch between the two different types of controllers. However, switching between controllers may be impractical when the vehicle network operate in a fully autonomous mode [9], [10]. Consequently, when no prior information on the group reference is available to the followers, it is more practical if the two problems can be solved using a single control architecture.

Many formation stabilization controllers have been developed for nonholonomic mobile robots, such as uniform δ-persistently exciting (uδ-PE) controller based on time-varying feedback [11], [12], and discontinuous feedback [13]. Also, numerous formation tracking control approaches have been proposed in the literature such as [14], [15]. However, only few works have been performed on the *simultaneous* formation stabilization and tracking control problem. Using the distributed estimation strategy, the problem was solved for mobile robots in [16] in the sense of global uniform ultimate boundedness. In [17], a simultaneous formation stabilization and tracking controller was proposed for mobile robots using a cascade design. Recently, a uδ-PE time-varying controller was proposed to solve the simultaneous formation stabilization and tracking control problem for heterogeneous planar underactuated vehicle networks in [10]. However, one of the side effects of the uδ-PE controllers presented in [10]–[12], [17] is the undesired oscillating behavior of the trajectories. Moreover, due to the vanishing PE property, the convergence rate becomes much slower as the trajectory converges to the origin. On the other hand, the discontinuous feedback such as the one in [13] may be difficult to implement on vehicle systems, which makes such controllers less practical.

These shortcomings may be overcome by designing controllers in polar coordinates. In [5], a smooth time-invariant controller is proposed to steer the mobile robots to the origin based on a weak Lyapunov function in polar coordinates. However, the polar coordinates introduce singularity precisely at the origin, and only *asymptotic* attractivity is established using the invariance principle. Thus, no (expo-

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nential) convergence rate is guaranteed. Later, a sliding mode controller is proposed in [18] to solve the trajectory tracking problem of mobile robots in polar coordinates. Using a similar idea as in [18], a consensus tracking controller is proposed in [19] for multi-vehicle systems. Recently, a smooth time-invariant controller has been proposed in [20] to solve the consensus problem for mobile robot networks. However, the control strategies proposed in [18], [19] require global position information of each agent in the network.

B. Main Contributions and Outline

In this work, we develop a distributed leader-follower formation controller for nonholonomic mobile robot networks. The main contributions are summarized as follows:

1) We solve the *simultaneous formation regulation and tracking* control problem for mobile robot networks using a single time-invariant control architecture. The controller can be used for both tasks without prior information.
2) We establish global asymptotic stability (GAS) and local exponential stability (LES) for the origin of formation error systems *in polar coordinates*, and correspondingly, establish the *exponential attractivity* for the origin in Cartesian coordinates with the domain of attraction contained in an *arbitrarily large compact set*.
3) The proposed control law does not require any global position measurements of the followers, i.e., the controller requires only *local* motion information, which can be measured directly using the on-board sensors. Moreover, the control law is *continuous along trajectories*, relatively simple, and thus is practical for real-world applications.

The rest of this paper is organized as follows. Preliminaries and problem formulation are given in Section II. Section III presents the controller design and the stability analysis. Simulation examples are presented in Section IV. Finally, the concluding remarks are provided in Section V.

II. Preliminaries and Problem Statement

Notations: Let $\mathbb{R}^n$ denote the n-dimensional Euclidean space; $\mathbb{R}_{>0}$ the set of all positive real numbers; $|\cdot|$ the Euclidean norm of vectors in $\mathbb{R}^n$. Let $\mathcal{B}_r := \{x \in \mathbb{R}^n : |x| \leq r\}$. The function $\mathrm{sinc} : \mathbb{R} \to \mathbb{R}$ is defined for all $x \neq 0$ by $\mathrm{sinc}(x) = \sin(x)/x$, and for $x = 0$ by the limiting value, i.e., $\mathrm{sinc}(0) = 1$. Note that the function $\mathrm{sinc}(\cdot)$ is bounded and smooth everywhere on its domain. We use the bold and non-italicized subscript $\mathbf{i}$ to denote the index of an agent in the multi-agent systems considered in this paper.

A. Model of Mobile Robot Networks

Consider a group of $N+1$ nonholonomic two-wheeled mobile robots, where the mobile robots are numbered $\mathbf{i} = 0, 1, \ldots, N$ with $\mathbf{0}$ representing the *real* group leader and $\mathbf{1}, \ldots, \mathbf{N}$ the followers, as shown in Fig. 1. The kinematic model for the $\mathbf{i}$-th mobile robot is given by

$$\dot{x}_{\mathbf{i}} = v_{\mathbf{i}} \cos(\theta_{\mathbf{i}}), \;\; \dot{y}_{\mathbf{i}} = v_{\mathbf{i}} \sin(\theta_{\mathbf{i}}), \;\; \dot{\theta}_{\mathbf{i}} = \omega_{\mathbf{i}}, \qquad (1)$$

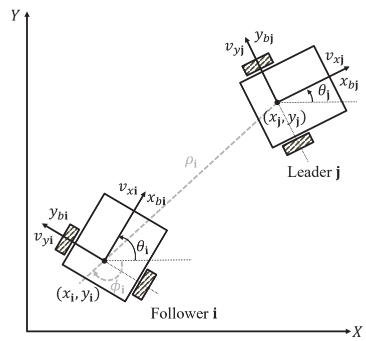

Fig. 1. Top view of the leader-follower formation of nonholonomic wheeled mobile robots.

where $(x_{\mathbf{i}}, y_{\mathbf{i}}) \in \mathbb{R}^2$ are the Cartesian coordinates of the center of mass of the robot, $\theta_{\mathbf{i}} \in \mathbb{R}$ is its orientation angle, the control inputs $v_{\mathbf{i}}$ and $\omega_{\mathbf{i}}$ are the linear and angular velocities of the mass center. Let us denote $q_{\mathbf{i}} := (x_{\mathbf{i}}, y_{\mathbf{i}}, \theta_{\mathbf{i}})$. We consider only the kinematic model in this work because once the desired velocities $v_{\mathbf{i}}(t)$ and $\omega_{\mathbf{i}}(t)$ are obtained, the actual control forces on the wheels can be calculated using backstepping technique or cascade approach based on the dynamic force-balanced equations.

The communication topology of the mobile robot network is modeled by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{\mathbf{0}, \mathbf{1}, \ldots, \mathbf{N}\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represent its sets of nodes and edges, respectively. The set of neighboring nodes with edges connected to the $\mathbf{i}$-th node is denoted by $\mathcal{N}_{\mathbf{i}} = \{\mathbf{j} \in \mathcal{V} : (\mathbf{i}, \mathbf{j}) \in \mathcal{E}\}$. The edges represent communication between the nodes such that node $\mathbf{i}$ obtains information from node $\mathbf{j}$, if $\mathbf{j} \in \mathcal{N}_{\mathbf{i}}$, as shown in Fig. 2. The constant weighted adjacency matrix $\mathcal{A} = [a_{\mathbf{ij}}]$ associated with $\mathcal{G}$ is defined according to the rule that $a_{\mathbf{ij}} > 0$ if $\mathbf{j} \in \mathcal{N}_{\mathbf{i}}$ and $a_{\mathbf{ij}} = 0$ otherwise. We assume graph $\mathcal{G}$ has no self-loop or loop and contains a directed spanning tree, which implies that there exists at least one directed path consisting of communication edges from the group leader to each follower in the network [8].

B. Problem Statement

We assume that the reference trajectory of the group leader is generated by the following *virtual* mobile robot

$$\dot{x}_{\mathtt{d}} = v_{\mathtt{d}} \cos(\theta_{\mathtt{d}}), \;\; \dot{y}_{\mathtt{d}} = v_{\mathtt{d}} \sin(\theta_{\mathtt{d}}), \;\; \dot{\theta}_{\mathtt{d}} = \omega_{\mathtt{d}}, \qquad (2)$$

where $(x_{\mathtt{d}}, y_{\mathtt{d}}, \theta_{\mathtt{d}})$ denotes the configuration of the virtual mobile robot, and $(v_{\mathtt{d}}, \omega_{\mathtt{d}})$ its linear and angular velocities. We make the following assumption on the reference trajectory of the group leader.

Assumption 1: The reference trajectory $(x_{\mathtt{d}}, y_{\mathtt{d}}, \theta_{\mathtt{d}}, v_{\mathtt{d}}, \omega_{\mathtt{d}})$ is only available to the group leader. Furthermore, the reference velocity $(v_{\mathtt{d}}(t), \omega_{\mathtt{d}}(t))$ is continuously differentiable, bounded. Moreover, the reference velocity $(v_{\mathtt{d}}(t), \omega_{\mathtt{d}}(t))$ satisfies *either* of the following mutually exclusive conditions:

A1) There exist T and $\mu_1 > 0$ such that

$$\int_t^{t+T} |\omega_{\mathtt{d}}(\tau)| \mathrm{d}\tau \geq \mu_1, \;\; \forall t \geq 0. \qquad (3)$$

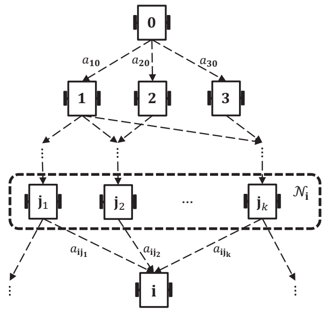

Fig. 2. Communication network graph of the mobile robot networks.

A2) There exist $\mu_2 > 0$ such that

$$\int_0^\infty (|v_d(\tau)| + |\omega_d(\tau)|)\,\mathrm{d}\tau \le \mu_2. \tag{4}$$

The objective of *formation regulation and tracking* is to design a distributed controller for each mobile robot such that it coordinates its motion relative to its neighbors, and the network *asymptotically converges* to a predefined geometric pattern under Assumption 1. The geometric pattern of the mobile robot network in terms of planar position is defined by a set of constant offset vectors $\{(d^x_{\mathbf{ij}}, d^y_{\mathbf{ij}}) \in \mathbb{R}^2 : \mathbf{i}, \mathbf{j} \in \mathcal{V}, \mathbf{i} \neq \mathbf{j}\}$. Specifically, under Assumption 1, we will design a control law $(v_\mathbf{i}, \omega_\mathbf{i})$ for each agent without global position measurements such that: i.) all states in the closed-loop system are bounded; ii.) all the mobile robots in the network maintain a prescribed formation in the sense that for all $\mathbf{i} \in \mathcal{V}$,

$$\lim_{t\to\infty} \sum_{\mathbf{j}\in\mathcal{N}_\mathbf{i}} \begin{vmatrix} x_\mathbf{i}(t) - x_\mathbf{j}(t) - d^x_{\mathbf{ij}} \\ y_\mathbf{i}(t) - y_\mathbf{j}(t) - d^y_{\mathbf{ij}} \\ \theta_\mathbf{i}(t) - \theta_\mathbf{j}(t) \end{vmatrix} = 0. \tag{5}$$

Remark 1: In Assumption 1, A1 implies that $\omega_d(\cdot)$ is PE, and A2 implies that the reference velocity (v_d, ω_d) belongs to $\mathcal{L}_1$ space, which means that the reference velocity converge to zero sufficiently fast. Note that the formation regulation and tracking problem covers two important cases:

Case 1 (Formation Regulation): If $v_d(t) \equiv \omega_d(t) \equiv 0$, then the formation regulation and tracking problem is reduced to the *formation regulation problem*.

Case 2 (Formation Tracking): If $\lim_{t\to\infty}[v_d^2(t) + \omega_d^2(t)] \neq 0$, then the formation regulation and tracking problem is reduced to the *formation tracking control problem*.

III. Formation Control Design

A. Error Dynamics in Polar Coordinates

For more compact notations, we denote the reference trajectory for each $\mathbf{i}$-th follower by

$$\begin{bmatrix} \bar{x}_\mathbf{i}(t) \\ \bar{y}_\mathbf{i}(t) \\ \bar{\theta}_\mathbf{i}(t) \end{bmatrix} := \frac{1}{\sum_{\mathbf{j}\in\mathcal{N}_\mathbf{i}} a_{\mathbf{ij}}} \sum_{\mathbf{j}\in\mathcal{N}_\mathbf{i}} a_{\mathbf{ij}} \begin{bmatrix} x_\mathbf{j}(t) + d^x_{\mathbf{ij}} \\ y_\mathbf{j}(t) + d^y_{\mathbf{ij}} \\ \theta_\mathbf{j}(t) \end{bmatrix}, \tag{6}$$

and denote the reference velocity by

$$\begin{bmatrix} \bar{v}_\mathbf{i}(t) \\ \bar{\omega}_\mathbf{i}(t) \end{bmatrix} := \frac{1}{\sum_{\mathbf{j}\in\mathcal{N}_\mathbf{i}} a_{\mathbf{ij}}} \sum_{\mathbf{j}\in\mathcal{N}_\mathbf{i}} a_{\mathbf{ij}} \begin{bmatrix} v_\mathbf{j}(t) \\ \omega_\mathbf{j}(t) \end{bmatrix}. \tag{7}$$

We will use the errors $\tilde{x}_\mathbf{i} = x_\mathbf{i} - \bar{x}_\mathbf{i}$, $\tilde{y}_\mathbf{i} = y_\mathbf{i} - \bar{y}_\mathbf{i}$, and $\tilde{\theta}_\mathbf{i} = \theta_\mathbf{i} - \bar{\theta}_\mathbf{i}$ in the control design, and only relative measurements are needed to construct these signals.

For each agent, we define the position formation error in polar coordinates as

$$\rho_\mathbf{i} := |(x_\mathbf{i} - \bar{x}_\mathbf{i}, y_\mathbf{i} - \bar{y}_\mathbf{i})|, \tag{8a}$$

$$\phi_\mathbf{i} := \operatorname{atan2}(y_\mathbf{i} - \bar{y}_\mathbf{i}, x_\mathbf{i} - \bar{x}_\mathbf{i}), \quad \forall \rho_\mathbf{i} > 0, \tag{8b}$$

where $\rho_\mathbf{i}$ and $\phi_\mathbf{i}$ denote the distance and the polar angle between the agent itself and its reference trajectory, respectively. The polar angle $\phi_\mathbf{i}$ can also be obtained directly using the line-of-sight angle and the orientation angle, as shown in Fig. 1. Note that the transformation (8a)-(8b) maps the formation error $(\tilde{x}_\mathbf{i}, \tilde{y}_\mathbf{i}, \tilde{\theta}_\mathbf{i}) \in \mathbb{R}^2\backslash\{(0,0)\} \times \mathbb{R}$ into polar coordinates $(\rho_\mathbf{i}, \phi_\mathbf{i}, \tilde{\theta}_\mathbf{i}) \in \mathbb{R}_{>0} \times \mathbb{R}^2$. Indeed, $(\rho_\mathbf{i}, \tilde{\theta}_\mathbf{i}) = (0, 0)$ is equivalent to $(\tilde{x}_\mathbf{i}, \tilde{y}_\mathbf{i}, \tilde{\theta}_\mathbf{i}) = (0, 0, 0)$. Thus, the error states in polar coordinates converging to zero implies the error states in Cartesian coordinates also converging to zero, which implies the formation objective (5) is achieved.

Taking time derivative of (8a)-(8b) along trajectories of (1), the error dynamics in polar coordinates are given by

$$\dot{\rho}_\mathbf{i} = v_\mathbf{i}\cos(\theta_\mathbf{i} - \phi_\mathbf{i}) - \bar{v}_\mathbf{i}\cos(\bar{\theta}_\mathbf{i} - \phi_\mathbf{i}), \tag{9a}$$

$$\dot{\phi}_\mathbf{i} = \frac{1}{\rho_\mathbf{i}}\left[v_\mathbf{i}\sin(\theta_\mathbf{i} - \phi_\mathbf{i}) - \bar{v}_\mathbf{i}\sin(\bar{\theta}_\mathbf{i} - \phi_\mathbf{i})\right], \tag{9b}$$

$$\dot{\tilde{\theta}}_\mathbf{i} = \omega_\mathbf{i} - \bar{\omega}_\mathbf{i}. \tag{9c}$$

Remark 2: The *formation stabilization* problem studied in [10], [17] requires the origin of the error system to be *asymptotically stable*. However, it is noted that the mapping $(\rho_\mathbf{i}, \phi_\mathbf{i}, \theta_\mathbf{i}) \mapsto (\tilde{x}_\mathbf{i}, \tilde{y}_\mathbf{i}, \tilde{\theta}_\mathbf{i})$ is not a diffeomorphism. Hence, the asymptotic stability of the origin in the polar coordinates may not be mapped into Cartesian coordinates. In this case, only *attractivity* is guaranteed for the origin in Cartesian coordinates. Thus, in contrast with *formation stabilization*, we only focus on *formation regulation* in this work, where only *attractivity* is required in the formation control objective.

B. Formation Control Design and Stability Analysis

To begin with, we first consider the regulation problem for each vehicle in the network, i.e., $\bar{v}_\mathbf{i}(t) \equiv \bar{\omega}_\mathbf{i}(t) \equiv 0$. Without loss of generality, we assume that the desired group orientation $\theta_d \equiv 0$. In this case, the error equations (9a)-(9c) reduce to

$$\dot{\rho}_\mathbf{i} = v_\mathbf{i}\cos\alpha_\mathbf{i}, \quad \dot{\phi}_\mathbf{i} = \frac{v_\mathbf{i}}{\rho_\mathbf{i}}\sin\alpha_\mathbf{i}, \quad \dot{\alpha}_\mathbf{i} = \omega_\mathbf{i} - \frac{v_\mathbf{i}}{\rho_\mathbf{i}}\sin\alpha_\mathbf{i}, \tag{10}$$

where $\alpha_\mathbf{i} := \tilde{\theta}_\mathbf{i} - \phi_\mathbf{i}$. Consider the following Lyapunov function candidate

$$V_\mathbf{i}(\rho_\mathbf{i}, \phi_\mathbf{i}, \alpha_\mathbf{i}) = \frac{1}{2}\left(\lambda_1\rho_\mathbf{i}^2 + \lambda_2\phi_\mathbf{i}^2 + \alpha_\mathbf{i}^2\right), \tag{11}$$

where λ_1, λ_2 are positive constants. The time derivative of $V_\mathbf{i}$ along trajectories of (10) is given by

$$\dot{V}_\mathbf{i} = \lambda_1\rho_\mathbf{i}v_\mathbf{i}\cos\alpha_\mathbf{i} + \lambda_2\phi_\mathbf{i}\frac{v_\mathbf{i}}{\rho_\mathbf{i}}\sin\alpha_\mathbf{i} + \alpha_\mathbf{i}\left(\omega_\mathbf{i} - \frac{v_\mathbf{i}}{\rho_\mathbf{i}}\sin\alpha_\mathbf{i}\right).$$

Consider the following control input

$$v_{\mathtt{i}} = -k_{1\mathtt{i}}\rho_{\mathtt{i}}\cos\alpha_{\mathtt{i}}, \tag{12}$$
$$\omega_{\mathtt{i}} = -k_{2\mathtt{i}}\alpha_{\mathtt{i}} - k_{1\mathtt{i}}\,\mathrm{sinc}(2\alpha_{\mathtt{i}})(\alpha_{\mathtt{i}} - \lambda_2\phi_{\mathtt{i}}), \tag{13}$$

where $k_{1\mathtt{i}}, k_{2\mathtt{i}}$ are positive control gains. In this case, it follows that for the closed-loop error system (10), (12), (13) given by

$$\dot{\rho}_{\mathtt{i}} = -k_{1\mathtt{i}}\cos^2(\alpha_{\mathtt{i}})\rho_{\mathtt{i}}, \tag{14a}$$
$$\dot{\phi}_{\mathtt{i}} = -k_{1\mathtt{i}}\,\mathrm{sinc}(2\alpha_{\mathtt{i}})\alpha_{\mathtt{i}}, \tag{14b}$$
$$\dot{\alpha}_{\mathtt{i}} = -k_{2\mathtt{i}}\alpha_{\mathtt{i}} + \lambda_2 k_{1\mathtt{i}}\,\mathrm{sinc}(2\alpha_{\mathtt{i}})\phi_{\mathtt{i}}, \tag{14c}$$

the time derivative of $V_{\mathtt{i}}(\cdot)$ along the trajectories of (14a)-(14c) satisfies

$$\dot{V}_{\mathtt{i}} = -\lambda_1 k_{1\mathtt{i}}\cos^2(\alpha_{\mathtt{i}})\rho_{\mathtt{i}}^2 - k_{2\mathtt{i}}\alpha_{\mathtt{i}}^2 \le 0. \tag{15}$$

Note that $(\rho_{\mathtt{i}}, \phi_{\mathtt{i}}, \alpha_{\mathtt{i}}) \to 0$ implies that $(\rho_{\mathtt{i}}, \phi_{\mathtt{i}}, \tilde{\theta}_{\mathtt{i}}) \to 0$. We have the following result.

Proposition 1: Consider the system (14a)-(14c). If $k_{1\mathtt{i}}, k_{2\mathtt{i}}$ and λ_2 are positive constants, then the origin $(\rho_{\mathtt{i}}, \phi_{\mathtt{i}}, \alpha_{\mathtt{i}}) = (0,0,0)$ is GAS. Furthermore, the trajectories of (14a)-(14c) with initial conditions contained in the compact set $\mathcal{B}_r$ exponentially converge to the origin for each $r > 0$.

Proof: It follows from (15) that the origin is globally stable. Note that $\dot{V}_{\mathtt{i}} \equiv 0$ implies that $\rho_{\mathtt{i}} \equiv \alpha_{\mathtt{i}} \equiv 0$, and it follows from (14c) that the largest invariant set contained in the set where $\dot{V}_{\mathtt{i}} \equiv 0$ is a point $\rho_{\mathtt{i}} = \alpha_{\mathtt{i}} = \phi_{\mathtt{i}} = 0$. Thus, it follows from the Krasovskii-LaSalle's invariance principle that the origin is GAS.

To see the exponential convergence of the trajectory in any compact set $\mathcal{B}_r$, we write (14b)-(14c) in the following form

$$\begin{bmatrix}\dot{\alpha}_{\mathtt{i}}\\ \dot{\phi}_{\mathtt{i}}\end{bmatrix} = \underbrace{\begin{bmatrix}-k_{2\mathtt{i}} & \lambda_2 k_{1\mathtt{i}}\\ -k_{1\mathtt{i}} & 0\end{bmatrix}}_{\mathcal{A}}\begin{bmatrix}\alpha_{\mathtt{i}}\\ \phi_{\mathtt{i}}\end{bmatrix} + \underbrace{\begin{bmatrix}-\lambda_2 k_{1\mathtt{i}}\left(1-\,\mathrm{sinc}(2\alpha_{\mathtt{i}})\right)\phi_{\mathtt{i}}\\ k_{1\mathtt{i}}\left(1-\,\mathrm{sinc}(2\alpha_{\mathtt{i}})\right)\alpha_{\mathtt{i}}\end{bmatrix}}_{K(\alpha_{\mathtt{i}},\phi_{\mathtt{i}})}.$$

The matrix $\mathcal{A}$ is Hurwitz, which implies there exist matrices $P = P^\top > 0$ and $Q = Q^\top > 0$ such that $\mathcal{A}^\top P + P\mathcal{A} = -Q$. Denote $\xi = [\alpha_{\mathtt{i}}, \phi_{\mathtt{i}}]^\top$, and it follows from the property of $\mathrm{sinc}(\cdot)$ function and the global boundedness of trajectories that for any $r > 0$ and for all $|\xi| < r$, we have $|K(\alpha_{\mathtt{i}}, \phi_{\mathtt{i}})| \le \kappa(|\xi|)|\alpha_{\mathtt{i}}|$, where $\kappa(\cdot)$ is a bounded non-decreasing function. Then, the total derivative of $W = \xi^\top P\xi$ along the trajectories of (14b)-(14c) is given as

$$\begin{aligned}\dot{W} &= -\xi^\top Q\xi + 2K^\top P\xi\\ &\le -\xi^\top Q\xi + 2\lambda_{\max}\{P\}\kappa(|\xi|)|\alpha_{\mathtt{i}}||\xi|\\ &\le -\xi^\top Q\xi + \lambda_{\max}\{P\}\left(\frac{\kappa(r)^2|\alpha_{\mathtt{i}}|^2}{\varepsilon} + \varepsilon|\xi|^2\right).\end{aligned}$$

where $\varepsilon > 0$ can be chosen arbitrarily small. Choosing $\varepsilon = \frac{1}{2}\lambda_{\min}\{Q\}/\lambda_{\max}\{P\}$ yields $\dot{W} \le -\frac{1}{2}\lambda_{\min}\{Q\}|\xi|^2 + 2\kappa(r)^2(\lambda_{\max}\{P\}^2/\lambda_{\min}\{Q\})|\alpha_{\mathtt{i}}|^2$. Consider the function

$$\mathcal{W}_r(\xi) = W(\xi) + \frac{\nu_r}{2}\left(\lambda_2\phi_{\mathtt{i}}^2 + \alpha_{\mathtt{i}}^2\right),$$

where $\nu_r = 2\kappa(r)^2(\lambda_{\max}^2\{P\}/\lambda_{\min}\{Q\})/k_{2\mathtt{i}}$. Then, for any $r > 0$ and for all $|\xi| < r$, we have $\dot{\mathcal{W}}_r \le -\frac{1}{2}\lambda_{\min}\{Q\}|\xi|^2$. Note that $\mathcal{W}_r(\xi)$ is a quadratic *strict* Lyapunov function for (14b)-(14c) on *any* compact set $\mathcal{B}_r$. Thus, we conclude exponential convergence of trajectories of (14b)-(14c) on *any* compact set $\mathcal{B}_r$. The exponential convergence of trajectories of (14a) can be easily obtained by rewriting (14a) into $\dot{\rho}_{\mathtt{i}} = -k_{1\mathtt{i}}\rho_{\mathtt{i}} + k_{1\mathtt{i}}(1-\cos^2(\alpha_{\mathtt{i}}))\rho_{\mathtt{i}}$, and considering the last term as an exponential vanishing disturbance (since $\rho_{\mathtt{i}}$ is bounded) which completes the proof. ■

Remark 3: Although we prove the GAS for the origin of (14a)-(14c), the error system is defined only for $\rho_{\mathtt{i}} > 0$. To deal with the case $\rho_{\mathtt{i}} = 0$, which is equivalent to $(\tilde{x}_{\mathtt{i}}, \tilde{y}_{\mathtt{i}}) = (0,0)$, we switch the control law to

$$v_{\mathtt{i}} = 0,\ \ \omega_{\mathtt{i}} = -k_{2\mathtt{i}}\tilde{\theta}_{\mathtt{i}} - k_{1\mathtt{i}}\,\mathrm{sinc}(2\tilde{\theta}_{\mathtt{i}})\tilde{\theta}_{\mathtt{i}}, \tag{16}$$

when $\rho_{\mathtt{i}} = 0$. The GAS and exponential convergence also can be proved using the strict Lyapunov function $\mathcal{W}_r$ constructed in the proof of Proposition 1. With the common Lyapunov function $\mathcal{W}_r$, the closed-loop system is GAS with trajectories exponentially converging to the origin under the switching rule. It is noted that although the controller is switching, it is continuous along trajectories.

Proposition 1 can be applied to the global leader **0** in the formation regulation problem, i.e., $v_{\mathtt{d}}(t) \equiv \omega_{\mathtt{d}}(t) \equiv 0$. Due to the hierarchy structure of the communication topology, the information of the reference signal is transformed to the followers. Thus, for the followers in the formation regulation, the reference velocity $(\bar{v}_{\mathtt{i}}, \bar{\omega}_{\mathtt{i}})$ may only exponentially converge to the zero. In this case, under the control law (12)-(13) the error system is given by

$$\begin{aligned}\dot{\rho}_{\mathtt{i}} &= -k_{1\mathtt{i}}\cos^2(\alpha_{\mathtt{i}})\rho_{\mathtt{i}} - \bar{v}_{\mathtt{i}}\cos(\bar{\theta}_{\mathtt{i}} - \phi_{\mathtt{i}}),\\ \dot{\phi}_{\mathtt{i}} &= -k_{1\mathtt{i}}\,\mathrm{sinc}(2\alpha_{\mathtt{i}})\alpha_{\mathtt{i}} - \frac{\bar{v}_{\mathtt{i}}}{\rho_{\mathtt{i}}}\sin(\bar{\theta}_{\mathtt{i}} - \phi_{\mathtt{i}}),\\ \dot{\alpha}_{\mathtt{i}} &= -k_{2\mathtt{i}}\alpha_{\mathtt{i}} + \lambda_2 k_{1\mathtt{i}}\,\mathrm{sinc}(2\alpha_{\mathtt{i}})\phi_{\mathtt{i}} + \frac{\bar{v}_{\mathtt{i}}}{\rho_{\mathtt{i}}}\sin(\bar{\theta}_{\mathtt{i}} - \phi_{\mathtt{i}}) - \bar{\omega}_{\mathtt{i}}.\end{aligned} \tag{17}$$

Proposition 2: Consider the system (9a)-(9c). Select the control law (12)-(13) when $\rho_{\mathtt{i}} > 0$, and switch it to (16) when $\rho_{\mathtt{i}} = 0$. If $k_{1\mathtt{i}}, k_{2\mathtt{i}}$ and λ_2 are positive constants and the reference velocities $\bar{v}_{\mathtt{i}}(t)$ and $\bar{\omega}_{\mathtt{i}}(t)$ converge to zero exponentially, then the origin $(\rho_{\mathtt{i}}, \phi_{\mathtt{i}}, \alpha_{\mathtt{i}}) = (0,0,0)$ is GAS. Furthermore, the trajectories of the closed-loop system with initial conditions contained in the compact set $\mathcal{B}_r$ exponentially converge to the origin for each $r > 0$.

Proof: We first prove the input-to-state stability (ISS) of the $(\alpha_{\mathtt{i}}, \phi_{\mathtt{i}})$-subsystem in (17) by viewing the reference velocity terms as inputs. For any $\delta > 0$ and for any $\rho_{\mathtt{i}} \ge \delta$, we have $1/\rho_{\mathtt{i}} \le 1/\delta$. Consider the $(\alpha_{\mathtt{i}}, \phi_{\mathtt{i}})$-dynamics. It follows from the proof of Proposition 1 that the nominal part (14b)-(14c) has a quadratic strict Lyapunov function $\mathcal{W}_r(\xi)$ on *arbitrarily large* compact set $\mathcal{B}_r$, which satisfies $|\partial\mathcal{W}_r(\xi)/\partial\xi| \le c|\xi|$ with $c > 0$. Evaluating the time derivative of $\mathcal{W}_r(\xi)$ along the trajectory of (17), yields

$$\begin{aligned}\dot{\mathcal{W}}_r &\le -\frac{1}{2}\lambda_{\min}\{Q\}|\xi|^2 + \left|\frac{\partial\mathcal{W}_r}{\partial\xi}\right|\left|\begin{bmatrix}-\frac{\bar{v}_{\mathtt{i}}}{\rho_{\mathtt{i}}}\sin(\bar{\theta}_{\mathtt{i}} - \phi_{\mathtt{i}})\\ \frac{\bar{v}_{\mathtt{i}}}{\rho_{\mathtt{i}}}\sin(\bar{\theta}_{\mathtt{i}} - \phi_{\mathtt{i}}) - \bar{\omega}_{\mathtt{i}}\end{bmatrix}\right|\\ &\le -\frac{1}{2}\lambda_{\min}\{Q\}|\xi|^2 + \sqrt{2}c|\xi|\left(\frac{|\bar{v}_{\mathtt{i}}|}{\delta} + |\bar{\omega}_{\mathtt{i}}|\right).\end{aligned} \tag{18}$$

Then, for $0 < \eta < 1$, we have $\dot{\mathcal{W}}_r \leq -\frac{1}{2}\lambda_{\min}\{Q\}(1-\eta)|\xi|^2$, for all $|\xi| \geq 2\sqrt{2}c(|\bar{v}_i|/\delta + |\bar{\omega}_i|)/(\lambda_{\min}\{Q\}\eta)$, which shows that (α_i, ϕ_i)-subsystem is ISS with respect to $(|\bar{v}_i|/\delta + |\bar{\omega}_i|)$. Then, GAS and exponential convergence of trajectories come from the the exponential convergence of the input $(|\bar{v}_i|/\delta + |\bar{\omega}_i|)$. The GAS and exponential convergence for ρ_i-subsystem come from rewriting the equation into $\dot{\rho}_i = -k_{1i}\rho_i + k_{1i}(1-\cos^2(\alpha_i))\rho_i - \bar{v}_i\cos(\bar{\theta}_i - \phi_i)$ and viewing the last two terms as exponentially vanishing inputs. In the case of $\rho_i = 0$, the ISS and exponential convergence can be easily shown using the same $\mathcal{W}_r(\xi)$. With the common Lyapunov function $\mathcal{W}_r$, the closed-loop system is GAS with trajectories exponentially converging to the origin under the switching rule. ■

While Proposition 2 can be used to solve the formation regulation problem, next we consider the formation tracking problem. Here, we first assume that for each agent, the reference angular velocity $\bar{\omega}_i(t)$ is PE, i.e., there exist T_i and $\mu_{1i} > 0$ such that

$$\int_t^{t+T_i} |\bar{\omega}_i(\tau)|\mathrm{d}\tau \geq \mu_{1i}, \ \forall t \geq 0. \tag{19}$$

We consider the following control law when $\rho_i > 0$,

$$v_i = \bar{v}_i(t) - k_{1i}\rho_i\cos\alpha_i, \tag{20}$$

$$\omega_i = \bar{\omega}_i(t) - k_{2i}\tilde{\theta}_i, \tag{21}$$

which yields the closed-loop system

$$\begin{aligned}
\dot{\rho}_i &= -k_{1i}\cos^2(\theta_i - \phi_i)\rho_i + \bar{v}_i\left[\cos(\theta_i - \phi_i) - \cos(\bar{\theta}_i - \phi_i)\right], \\
\dot{\phi}_i &= -\frac{k_{1i}}{2}\sin[2(\theta_i - \phi_i)] - \frac{\bar{v}_i}{\rho_i}\left[\sin(\theta_i - \phi_i) - \sin(\bar{\theta}_i - \phi_i)\right], \\
\dot{\tilde{\theta}}_i &= -k_{2i}\tilde{\theta}_i. \qquad \text{(22a)-(22c)}
\end{aligned}$$

When $\rho_i = 0$, the control law is switched to

$$v_i = \bar{v}_i(t), \ \ \omega_i = \bar{\omega}_i(t) - k_{2i}\tilde{\theta}_i. \tag{23}$$

Proposition 3: Consider the system (9a)-(9c). Select the control law (20)-(21) when $\rho_i > 0$, and switch it to (23) when $\rho_i = 0$. Assume that (19) holds. If $0 < k_{1i} < 2\mu_{1i}/T_i$ and $k_{2i} > 0$, then $|(\rho_i(t), \tilde{\theta}_i(t))| \to 0$ exponentially as $t \to \infty$.

Proof: For any $\delta > 0$ and for any $\rho_i \geq \delta$, we have $1/\rho_i \leq 1/\delta$. It is obvious from (22c) that $\tilde{\theta}_i(t) \to 0$ exponentially as $t \to \infty$, and, using basic trigonometric identities, it follows that $\cos(\theta_i - \phi_i) - \cos(\bar{\theta}_i - \phi_i)$ and $\sin(\theta_i - \phi_i) - \sin(\bar{\theta}_i - \phi_i)$ converge to zero exponentially as $t \to \infty$. Then, note that for the system $\dot{\rho}_i = -k_{1i}\cos^2(\theta_i(t) - \phi_i(t))\rho_i$, if $\cos(\theta_i(t) - \phi_i(t))$ is PE, then $\rho_i(t) \to 0$ exponentially as $t \to \infty$ [10]. Next, it follows from (22b) that $|\dot{\phi}_i(t)| \leq k_{1i}/2 < \mu_{1i}/T_i$ after a finite time T_f. The condition (19) implies that $\frac{1}{T_i}\int_t^{t+T_i} |\dot{\bar{\theta}}_i(\tau)|\mathrm{d}\tau \geq \mu_{1i}/T_i, \ \forall t \geq 0$, which means that the average of the reference angular velocity is larger than μ_{1i}/T_i. Thus, $\cos(\theta_i(t) - \phi_i(t))$ is PE, and $\rho_i(t) \to 0$ exponentially as $t \to \infty$. ■

It is also noted that the switching between control laws (20)-(21), and (23) is continuous along trajectories, that is, control law (20)-(21) continuously reduces to (23) as $\rho_i(t) \to 0$.

Finally, to achieve the simultaneous formation regulation and tracking control, we define a time-varying signal $z_i(t)$ for each agent as the solution of the following differential equation

$$\dot{z}_i = -\left(|\bar{v}_i(t)| + |\bar{\omega}_i(t)|\right)z_i, \ \ z_i(0) = 1, \tag{24}$$

that is, $z_i(t) = e^{-\int_0^t(|\bar{v}_i(\tau)| + |\bar{\omega}_i(\tau)|)\mathrm{d}\tau}$, $\forall t \geq 0$. We consider the following control law when $\rho_i > 0$,

$$v_i = \bar{v}_i - k_{1i}\rho_i\cos\alpha_i, \tag{25}$$

$$\omega_i = \bar{\omega}_i - k_{2i}\tilde{\theta}_i - z_i\left(k_{2i}\alpha_i + k_{1i}\,\mathrm{sinc}(2\alpha_i)(\alpha_i - \lambda_2\phi_i)\right). \tag{26}$$

When $\rho_i = 0$, the control law is switched to

$$v_i = \bar{v}_i(t), \tag{27}$$

$$\omega_i = \bar{\omega}_i(t) - k_{2i}\tilde{\theta}_i - z_i k_{1i}\,\mathrm{sinc}(2\tilde{\theta}_i)\tilde{\theta}_i. \tag{28}$$

Note that if the reference angular velocity $\bar{\omega}_i(\cdot)$ is PE, then $z_i(t) \to 0$ exponentially as $t \to \infty$. If the reference velocity belongs to $\mathcal{L}_1$ space, i.e., $\int_0^\infty(|\bar{v}_i(\tau)| + |\bar{\omega}_i(\tau)|)\,\mathrm{d}\tau$ is finite, then $z_i(t)$ converges to a certain positive constant as $t \to \infty$.

Theorem 1: Consider a group of nonholonomic wheeled mobile robots satisfying Assumption 1. Then, the formation regulation and tracking control problem is solved using the control law (25)-(28), if the control gains are selected such that $0 < k_{1i} < 2\mu_{1i}/T_i$ and $k_{2i} > 0$, and if the directed communication graph $\mathcal{G}$ contains a spanning tree.

Proof: In the case of A1 of Assumption 1, that is, the reference angular velocity is PE, we have $z_i(t) \to 0$ exponentially, and the control law (25)-(28) reduces to (20)-(21) and (23). It follows from Proposition 3 that $|(\rho_i(t), \tilde{\theta}_i(t))| \to 0$ exponentially as $t \to \infty$. On the other hand, in the case of A2 of Assumption 1, that is, the reference velocity belongs to $\mathcal{L}_1$, we have $z_i(t)$ converging to some positive constant as $t \to \infty$. Then, the control law (25)-(28) reduces to (12)-(13) and (16). It follows from Proposition 2 that the error system is GAS and LES and $|(\rho_i(t), \tilde{\theta}_i(t))| \to 0$ exponentially as $t \to \infty$. Finally, if the communication graph contains a spanning tree, then the information from the leader passes down to an agent in the network which, in turn, sends its own information to the neighboring agents and so on, and thus, the convergence holds for all vehicles in the network, which implies that the control objective (5) is achieved. ■

IV. Numerical Simulations

Consider a group of six nonholonomic mobile robots. The communication topology and the weighted adjacency matrix are shown in Fig. 3. The desired geometric pattern in formation is assumed to be a regular hexagon with the side length $h = 2$, i.e., $(d_{10}^x, d_{10}^y) = (-1, -\sqrt{3})$, $(d_{21}^x, d_{21}^y) = (1, -\sqrt{3})$, $(d_{32}^x, d_{32}^y) = (0, 2)$, $(d_{43}^x, d_{43}^y) = (1, \sqrt{3})$, and $(d_{54}^x, d_{54}^y) = (-1, \sqrt{3})$. The vehicles are assumed to be initially stationary at the coordinates $q_0(0) = (0,0,0)$, $q_1(0) = (-5,-5,0)$, $q_2(0) = (-2,-7,1)$, $q_3(0) = (3,-5,0)$, $q_4(0) = (6,-4,2)$, $q_5(0) = (5,2,0)$.

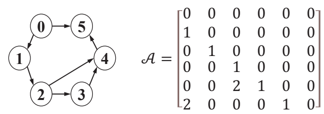

Fig. 3. Directed communication topology and the weighted adjacency matrix used in the simulations.

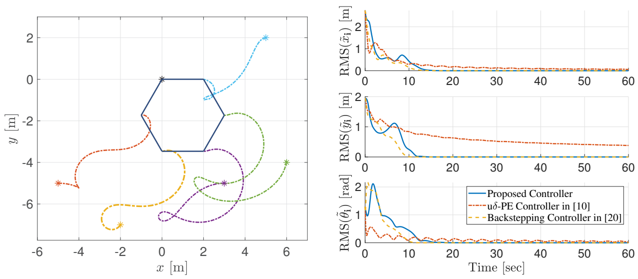

Fig. 4. Position paths in the plane and time history of the RMS error of the formation regulation.

1) Formation Regulation: In the first simulation, the desired configuration for the group leader **0** is at the origin for all times $t \geq 0$. The control parameters are selected as $k_{1\mathtt{i}} = k_{1\mathtt{i}} = \lambda_2 = 1$. The simulation results are shown in Fig. 4, where the root mean square (RMS) error is of the form $\mathrm{RMS}([\cdot]_{\mathtt{i}}) = (\frac{1}{n}\sum_{i=1}^{n}[\cdot]_{\mathtt{i}}^2)^{1/2}$. The comparison shows that the formation errors approach zero after 20 seconds using the proposed controller and the backstepping controller in [20], while the uδ-PE controller proposed in [10] converges after more than 60 seconds.

2) Formation Tracking: In the second simulation, the desired path for the group leader **0** is a harmonic function, i.e., $q_{\mathtt{d}}(0) = (0,0,0)$, $v_{\mathtt{d}}(t) = 1$ and $\omega_{\mathtt{d}}(t) = \cos t$ for all $t \geq 0$. The control parameters are selected as $k_{1\mathtt{i}} = k_{1\mathtt{i}} = \lambda_2 = 1$. The simulation results are shown in Fig. 5. It is observed that all formation tracking errors approach zero smoothly, while the trajectories with uδ-PE controller proposed in [10] show the undesired oscillatory behavior in the transient.

V. Conclusion

A distributed simultaneous regulation and tracking controller is proposed for nonholonomic wheeled mobile robot networks, based on the polar-coordinates model. Under the assumption of the network topology containing a directed spanning tree, the origin of the closed-loop system is GAS and LES in polar coordinates, and correspondingly, exponentially attractive in any compact set. The presented controller is time-invariant, continuous along trajectories, and does not require global position measurements of the followers.

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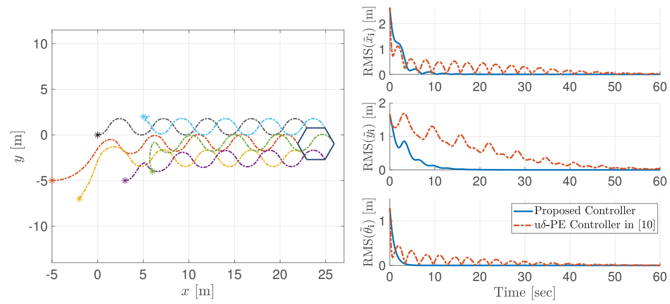

Fig. 5. Position paths in the plane and time history of the RMS error of the formation tracking.

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