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A Unified Approach to Stabilization, Trajectory Tracking, and Formation Control of Planar Underactuated Vehicles*

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Abstract— **This paper develops a procedure to design passivity-based controllers for stabilization, trajectory tracking, and formation control of a class of underactuated planar vehicles. The proposed approach uses generalized canonical transformations and energy shaping techniques and offers a framework to design stabilization and trajectory tracking controllers with the same structure. This approach is further applied to multi-agent formation stabilization and tracking problems. The proposed method can be used to solve motion control problems for a class of underactuated planar vehicles with a unified controller structure. As an example, the method is then applied to design controllers for planar mobile robots. Numerical simulations are presented for each case.**

I. INTRODUCTION

Underactuated mechanical systems are difficult to control because the control inputs have no direct influence on some variables at certain states [1]. High-gain designs, such as sliding mode control [2], [3], backstepping [4], [5], [6], although useful, have several disadvantages, such as sensitivity to measurement noise, saturation, singularities, and the reduction of robustness due to nonlinear cancellations [7].

Energy-based design methods have been shown to be effective and powerful in controlling underactuated mechanical systems since energy is a fundamental concept and has a clear meaning for these systems [8]. The control design problem for these systems can be recast as an energy-shaping problem such that the energy of the closed-loop system takes the desired form. This energy-based design approach is the essence of passivity-based techniques. Port-Hamiltonian framework is a suitable tool for energy/passivity-based control design of mechanical systems because of clearly defined characteristics such as energy dissipation and conservation [9]. Techniques such as generalized canonical transformation [10] and interconnection and damping assignment passivity-based control (IDA-PBC) [11] can be used to shape the energy of closed-loop systems to a desired form and preserve the port-Hamiltonian structure.

Despite the ease in employing passivity-based control design methodology in stabilization, it is difficult to adapt to trajectory tracking control design. Traditional approach in trajectory tracking control design is to construct and stabilize the error dynamics. However the error systems are usually time-varying due to the inclusion of the reference signals, making it difficult to be written into the port-Hamiltonian form. Consequently, a generalized canonical transformation [12] was suggested to transform the error dynamics into a port-Hamiltonian form and then stabilize the transformed systems [13]. The disadvantage of the application of such a transformation is that the clear energy concepts in mechanical systems become obscure.

In this paper, we propose a procedure to design passivity-based controllers for both stabilization and trajectory tracking control for a class of underactuated planar vehicles with zero potential energy such as mobile robots, surface vessels, and other forms of vehicles moving in the horizontal plane. In the stabilization design of underactuated planar vehicles, generalized canonical transformation is applied to shape the kinetic and potential energy of the system. In fact, generalized canonical transformation can be seen as an extension of IDA-PBC. Furthermore, in the trajectory tracking controller design, the feasible trajectories are generated using the nonholonomic constraints of the systems and then trajectory tracking is achieved by appropriate shaping of the system energy. Using this approach, stabilization and tracking control designs employ a unified framework and hence have the same control structure. Unlike the early work [13], the proposed method doesn't depend on the construction of error dynamics and thus has a clear physical meaning (energy) in the tracking control design. Numerical simulations are presented for each case.

Notation. All mappings are assumed smooth. I_n is the $n \times n$ identity matrix. For a full-rank mapping $G(x) \in \mathbb{R}^{n\times m}$ with $m < n$, we denote the generalized inverse $G^\dagger(x) := [G^\top(x)G(x)]^{-1}G^\top(x)$, and $G^\perp(x) \in \mathbb{R}^{(n-m)\times n}$ a full-rank left annihilator of mapping $G(x)$, i.e., $G^\perp(x)G(x) = 0$. In multi-agent context, agents are indexed as $i = 1,\ldots,N$, and the subscript $\mathbf{i}$ is bold and non-italicized.

II. PRELIMINARIES

This section briefly refers to the existing results on passivity-based stabilization of port-Hamiltonian mechanical systems.

A. Port-Hamiltonian Mechanical Systems

A time-varying input-state-output port-Hamiltonian system with state space manifold $\mathscr{X}$, input and output space $\mathscr{U} = \mathscr{Y}$, and Hamiltonian $H : \mathscr{X} \times \mathbb{R} \to \mathbb{R}$, is given as

$$\begin{aligned} \dot{x} &= [J(x,t) - R(x,t)]\frac{\partial H(x,t)}{\partial x}^\top + g(x,t)u \\ y &= g^\top(x,t)\frac{\partial H(x,t)}{\partial x}^\top, \end{aligned} \tag{1}$$

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where $J(x,t)=-J^\top(x,t)$ is the skew-symmetric interconnection structure and $R(x,t)=R^\top(x,t)\geq 0$ the non-negative resistive structure. A time-invariant port-Hamiltonian system is passive with storage function $H(x)$ if $H(x)$ is lower bounded [9]. However, a time-varying port-Hamiltonian system does not imply passivity [12].

A port-Hamiltonian formulation of standard mechanical systems is given as

$$\begin{bmatrix}\dot q\\ \dot p\end{bmatrix}=\begin{bmatrix}0 & J_{12}(q)\\ -J_{12}^\top(q) & J_{22}(q,p)-R_{22}(q,p)\end{bmatrix}\begin{bmatrix}\frac{\partial H}{\partial q}^\top\\ \frac{\partial H}{\partial p}^\top\end{bmatrix}+\begin{bmatrix}0\\ G(q)\end{bmatrix}u$$
$$y=G^\top(q)\frac{\partial H}{\partial p}^\top, \tag{2}$$

where $q\in\mathbb{R}^n$, $p\in\mathbb{R}^n$ are the generalized pose and momentum vectors, respectively, and $u\in\mathbb{R}^m$, $y\in\mathbb{R}^m$ are the control input and output variables, respectively. The Hamiltonian $H(q,p)$ is the total mechanical energy of the system

$$H(q,p)=\frac{1}{2}p^\top M^{-1}(q)p+V(q), \tag{3}$$

$M(q)=M^\top(q)>0$ is the inertia matrix and $V(q)$ is the potential energy function, $J_{12}(q)$ the kinematic transformation matrix, $J_{22}(q,p)=-J_{22}^\top(q,p)$ the internal interconnection matrix, $R_{22}(q,p)=R_{22}^\top(q,p)\geq 0$ the natural damping matrix, and $G(q)\in\mathbb{R}^{n\times m}$ is the input matrix with full rank. The mechanical system (2) is called fully-actuated if $m=n$, and underactuated if $m<n$. Mechanical systems with l nonholonomic constraints also can be written into port-Hamiltonian form (2) and in this case, $q\in\mathbb{R}^n$, $p\in\mathbb{R}^{n_p}$ with $n_p=n-l$ [9].

B. Generalized Canonical Transformation

Passivity-based control (PBC) is a framework for robust control design which achieves stabilization by passivation [14]. Since passivity is closely related to the concept of energy, it provides a natural design procedure to shape the energy of a system without the need for nonlinear cancellations and high-gain design. Generalized canonical transformation was proposed in [12] to change the system properties without changing the inherent port-Hamiltonian structure and thus can be used as a tool for stabilization. A set of transformations

$$\begin{aligned}\bar x&=\Phi(x,t)\\ \bar H&=H(x,t)+U(x,t)\\ \bar y&=y+\alpha(x,t)\\ \bar u&=u+\beta(x,t)\end{aligned} \tag{4}$$

that changes the state, Hamiltonian, output and input (x,H,y,u) to $(\bar x,\bar H,\bar y,\bar u)$ is said to be a generalized canonical transformation for system (1), if it preserves port-controlled Hamiltonian structure. Thus system in (1) can be transformed to

$$\begin{aligned}\dot{\bar x}&=[\bar J(\bar x,t)-\bar R(\bar x,t)]\frac{\partial \bar H(\bar x,t)}{\partial \bar x}^\top+\bar g(\bar x,t)\bar u\\ \bar y&=\bar g^\top(\bar x,t)\frac{\partial \bar H(\bar x,t)}{\partial \bar x}^\top.\end{aligned} \tag{5}$$

Since nonholonomic systems cannot be stabilized by smooth time-invariant controllers due to the Brockett's necessary condition not being satisfied, it is possible to stabilize a class of nonholonomic systems by time-varying generalized canonical transformations and passivity-based controllers [10]. The main result of [12] is contained in the proposition below.

Proposition 1: [12] (i) Consider the port-controlled Hamiltonian system (1). For any functions $U(x,t)$ and $\beta(x,t)$, there exists a pair of functions $\Phi(x,t)\in\mathbb{R}^n$ and $\alpha(x,t)\in\mathbb{R}^m$ such that the transformation (4) yields a generalized canonical transformation. Function Φ is the solution to the partial differential equation (PDE)

$$\frac{\partial\Phi}{\partial(x,t)}\begin{bmatrix}(J-R)\frac{\partial U}{\partial x}^\top+g\beta+(K-S)\frac{\partial(H+U)}{\partial x}^\top\\ -1\end{bmatrix}=0 \tag{6}$$

with $K(x,t)=-K^\top(x,t)\in\mathbb{R}^{n\times n}$ and $S(x,t)=S^\top(x,t)\in\mathbb{R}^{n\times n}$ satisfying $R+S\geq 0$. The change of output $\alpha(x,t)$ is given by

$$\alpha(x,t)=g^\top(x,t)\frac{\partial U}{\partial x}^\top(x,t). \tag{7}$$

Matrices $\bar J,\bar R,\bar g$ of the new port-controlled Hamiltonian system (5) are given by

$$\bar J=\frac{\partial\Phi}{\partial x}(J+K)\frac{\partial\Phi}{\partial x}^\top,\ \bar R=\frac{\partial\Phi}{\partial x}(R+S)\frac{\partial\Phi}{\partial x}^\top,\ \bar g=\frac{\partial\Phi}{\partial x}g. \tag{8}$$

(ii) If the system (1) is transformed using the generalized canonical transformation with (U,β) such that $\bar H=H+U\geq 0$, then the new input-output mapping $\bar u\mapsto\bar y$ is passive with the storage function $\bar H$ if and only if

$$\frac{\partial\bar H}{\partial(x,t)}\begin{bmatrix}(J-R)\frac{\partial U}{\partial x}^\top+g\beta-S\frac{\partial\bar H}{\partial x}^\top\\ -1\end{bmatrix}\geq 0. \tag{9}$$

(iii) Suppose moreover that (9) holds, that $H+U>0$, and the system (5) is zero state detectable (ZSD) with respect to the original state. Then the feedback $\bar u=-C(x,t)\bar y$ with $C(x,t)\geq\varepsilon I>0\in\mathbb{R}^{m\times m}$ renders the system (1) asymptotically stable. Suppose moreover that $H+U$ is decrescent and the transformed system is periodic, then the feedback renders the system uniformly asymptotically stable.

III. Main Results

Generalized canonical transformation is a powerful tool for stabilization of nonlinear systems, especially those which cannot be stabilized by smooth time-invariant controllers such as IDA-PBC. On the other hand, stabilization based on generalized canonical transformation has the same spirit of IDA-PBC, i.e. stabilizing the system by shaping its kinetic and potential energy. Therefore, stabilization based on generalized canonical transformation can be seen as an extension of IDA-PBC [11]. Furthermore, there is a version of the controller design based on generalized canonical transformation for tracking problems [13], which transforms the error dynamics into port-Hamiltonian systems and stabilizes the transformed system. Here we apply a similar transformation and design a uniformly asymptotically stabilizing control law for trajectory tracking of systems with zero potential energy.

Theorem 1: (i) Consider the port-controlled Hamiltonian system (2) with potential energy function $V(q)=0$. Assume there exists a periodic odd function (with respect to time) $\gamma:\mathbb{R}^n\times\mathbb{R}\in\mathbb{R}^{n_p}$ that satisfies the PDE

$$G^{\perp}\left[\frac{\partial\gamma(q,t)}{\partial q}J_{12}(q)M^{-\frac{1}{2}}p-(J_{22}-R_{22})M^{-\frac{1}{2}}\gamma(q,t)+M^{\frac{1}{2}}\frac{\partial\gamma(q,t)}{\partial t}\right]=0 \tag{10}$$

Then, the generalized canonical transformation

$$\begin{aligned}\begin{bmatrix}\bar q\\ \bar p\end{bmatrix}&=\Phi(q,p,t)=\begin{bmatrix}\Psi(q,t)\\ M^{-\frac{1}{2}}p+\gamma(q,t)\end{bmatrix}\\ \bar H&=H+p^\top M^{-\frac{1}{2}}\gamma(q,t)+\frac{1}{2}\gamma^\top(q,t)\gamma(q,t)\\ \bar y&=y+G^\top(q)M^{-\frac{1}{2}}\gamma(q,t)\\ \bar u&=u+G^\dagger(q)\left[\frac{\partial\gamma(q,t)}{\partial q}J_{12}(q)M^{-\frac{1}{2}}p-(J_{22}-R_{22})M^{-\frac{1}{2}}\gamma(q,t)+M^{\frac{1}{2}}\frac{\partial\gamma(q,t)}{\partial t}\right]\end{aligned} \tag{11}$$

transforms the system into a passive port-controlled Hamiltonian system

$$\begin{aligned}\begin{bmatrix}\dot{\bar q}\\ \dot{\bar p}\end{bmatrix}&=\begin{bmatrix}0 & \frac{\partial\Psi}{\partial q}J_{12}(q)M^{-\frac{1}{2}}\\ -M^{-\frac{1}{2}}J_{12}^\top(q)\frac{\partial\Psi}{\partial q}^\top & (J_{22}-R_{22})M^{-1}\end{bmatrix}\begin{bmatrix}\frac{\partial\bar H}{\partial\bar q}^\top\\ \frac{\partial\bar H}{\partial\bar p}^\top\end{bmatrix}+\begin{bmatrix}0\\ M^{-\frac{1}{2}}G(q)\end{bmatrix}\bar u\\ \bar y&=G(q)^\top M^{-\frac{1}{2}}\frac{\partial\bar H}{\partial\bar p}^\top\end{aligned} \tag{12}$$

where $(q,p)=\Phi^{-1}(\bar q,\bar p,t)$, the Hamiltonian

$$\bar H:=\frac{1}{2}\bar p^\top\bar p, \tag{13}$$

and the coordinate transformation $\bar q=\Psi(q,t)$ is a solution of the PDE

$$\frac{\partial\Psi(q,t)}{\partial(q,t)}\begin{bmatrix}J_{12}(q)M^{-\frac{1}{2}}\gamma(q,t)\\ -1\end{bmatrix}=0. \tag{14}$$

(ii) Furthermore, assume there exists a positive definite function $W:\mathbb{R}^n\to\mathbb{R}$ satisfying the PDE

$$G^{\perp}(q)\left[J_{12}^\top(q)\frac{\partial W(\Psi(q,t))}{\partial q}^\top\right]=0. \tag{15}$$

If the system (12) is zero state detectable with respect to the original state (q,p), then the generalized canonical transformation

$$\begin{aligned}\hat H&=\bar H+W(\bar q)\\ \hat u&=\bar u+G^\dagger(q)\left[J_{12}^\top(q)\frac{\partial W(\Psi(q,t))}{\partial q}^\top\right]\end{aligned} \tag{16}$$

with the feedback

$$\hat u=-C(q,p,t)\bar y \tag{17}$$

with $C(q,p,t)\ge\varepsilon I>0\in\mathbb{R}^{m\times m}$, renders the system (2) uniformly asymptotically stable.

Proof: The stabilization procedure is based on the passivation idea. The first step is to use a time-varying generalized canonical transformation to transform the original system into a (time-varying) passive port-controlled Hamiltonian system based on Proposition 1. Next, stabilize the system by interconnection of two passive subsystems (see e.g. [9], [7]) and by zero state detectable system assumption. We may then conclude the uniform asymptotic stability of the closed-loop system.

(i) For the part of shaping kinetic energy of the system, from Proposition 1, we have to use a generalized canonical transformation such that the new Hamiltonian $\bar H:=H+U\ge 0$. To make the new Hamiltonian $\bar H$ quadratic based on (3), let

$$U(q,p,t)=p^\top M^{-\frac{1}{2}}\gamma(q,t)+\frac{1}{2}\gamma^\top(q,t)\gamma(q,t) \tag{18}$$

and hence

$$\bar H=\frac{1}{2}\left(M^{-\frac{1}{2}}p+\gamma(q,t)\right)^\top\left(M^{-\frac{1}{2}}p+\gamma(q,t)\right)\ge 0. \tag{19}$$

Since the change of output in the generalized canonical transformation, as presented in (7), is only related to U, it is derived as

$$\bar y=y+G^\top(q)M^{-\frac{1}{2}}\gamma(q,t). \tag{20}$$

Taking $S=0$, the passivity-preserving condition (9) reduces to

$$\left(M^{-\frac{1}{2}}p+\gamma\right)^\top\begin{bmatrix}\frac{\partial\gamma}{\partial q} & M^{-\frac{1}{2}} & \frac{\partial\gamma}{\partial t}\end{bmatrix}\cdot\begin{bmatrix}J_{12}M^{-\frac{1}{2}}\gamma\\ -J_{12}^\top\frac{\partial\gamma}{\partial q}^\top\left(M^{-\frac{1}{2}}p+\gamma\right)+(J_{22}-R_{22})M^{-\frac{1}{2}}\gamma+G\beta\\ -1\end{bmatrix}\ge 0. \tag{21}$$

Note that if β is such that

$$G(q)\beta=\frac{\partial\gamma(q,t)}{\partial q}J_{12}(q)pM^{-\frac{1}{2}}p-(J_{22}-R_{22})M^{-\frac{1}{2}}\gamma(q,t)+M^{\frac{1}{2}}\frac{\partial\gamma(q,t)}{\partial t} \tag{22}$$

then (21) holds and β can be obtained a solution to (22). By the assumption of (10), the change of input can be obtained as in (11). From (6), the coordinate transformation can be obtained by setting

$$K = \begin{bmatrix} 0 & 0 \\ 0 & J_{12}^{\top}(q)\frac{\partial \gamma}{\partial q}^{\top} M^{\frac{1}{2}} - M^{\frac{1}{2}}\frac{\partial \gamma}{\partial q} J_{12}(q) \end{bmatrix}. \quad (23)$$

It follows from Proposition 1 of [15] that $\gamma(q,t)$ being periodic odd function with respect to time guarantees that (14) has a solution for $\Psi(q,t)$.
(ii) For the part of shaping potential energy of the system, it is easy to change the passive system (12) to another passive Hamiltonian system with a positive definite function by adding a positive potential function $W(\bar{q})$ and obtaining the corresponding generalized canonical transformation (16). □

IV. APPLICATION TO MOBILE ROBOTS

A schematic figure of a nonholonomic two-wheeled mobile robot is shown in Fig.1. The robot is subjected to the nonholonomic constraint of no-slip in the normal direction to the motion path. The dynamics of the mobile robot are given as [16]

$$\begin{cases} \dot{x} = v_x \cos\theta - d\omega \sin\theta \\ \dot{y} = v_x \sin\theta + d\omega\cos\theta \\ \dot{\theta} = \omega \\ \dot{v}_x = \frac{md}{\tilde{m}}\omega^2 + \frac{1}{\tilde{m}r}(\tau_L + \tau_R) \\ \dot{\omega} = -\frac{md}{\tilde{I}}\omega v_x + \frac{l}{2\tilde{I}r}(\tau_R - \tau_L), \end{cases} \quad (24)$$

where (x,y) is the position of the vehicle's mass center, θ is the orientation, v_x, v_y are the projections of the velocity of the mass center onto the body-fixed frame $x_b y_b$, and τ_L, τ_R are differential torques applied to each wheel respectively. Also m is the total mass of the vehicle, I is the moment of inertia of the vehicle about the axis orthogonal to the plane passing through the mass center, $\tilde{m} := m + 2J/r^2$ and $\tilde{I} := I + md^2 + (l^2/r^2)J$, and J is the moment of inertia of the wheels about their axis of rotation. By a change of input $u_1 = (\tau_L + \tau_R)/r, u_2 = l(\tau_R - \tau_L)/2r$, the dynamics (24) can be written in a standard port-controlled Hamiltonian form (2) with states $q = [x, y, \theta]^{\top}, p = M[v_x, \omega]^{\top}$ and control input $u = [u_1, u_2]^{\top}$. The matrices in the system are given by

$$J_{12}(q) = \begin{bmatrix} \cos q_3 & -d\sin q_3 \\ \sin q_3 & d\cos q_3 \\ 0 & 1 \end{bmatrix}, J_{22}(p) = \begin{bmatrix} 0 & \frac{md p_2}{\tilde{I}} \\ -\frac{md p_2}{\tilde{I}} & 0 \end{bmatrix},$$

$M = \mathrm{diag}\{\tilde{m}, \tilde{I}\}$, $R_{22} = 0, G(q) = I_2$ and $V(q) = 0$.

A. Stabilization of a single robot

Since mobile robots (24) are nonholonomic systems, a time-varying control law must be considered. The stabilization controller of a single mobile robot (24) can be derived directly from Proposition 1 using a time-varying generalized canonical transformation [10].

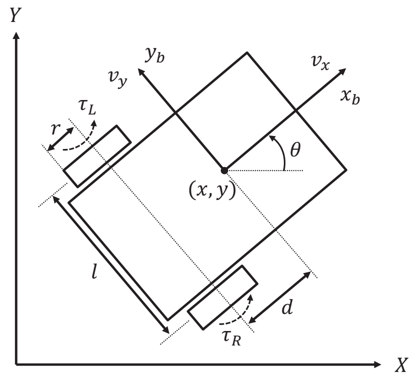

Fig. 1. A nonholonomic two-wheeled differential drive mobile robot.

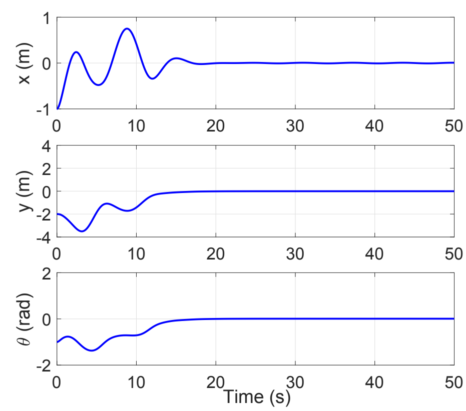

Fig. 2. Stabilization of mobile robot's pose vector $q = [x, y, \theta]^{\top}$.

Specifically, using Theorem 1, choose

$$\gamma(q,t) = \begin{bmatrix} q_3 \sin t \\ 0 \end{bmatrix}, W(\bar{q}) = \frac{1}{2}\bar{q}^{\top}\bar{q}. \quad (25)$$

Note that for the mobile robot (24), $G = I_2$ and hence, $G^{\perp} = 0$, which implies that condition (10) holds. Thus, the generalized canonical transformations (11) and (16) can be obtained. We choose the gain matrix $C(q,p,t) = \mathrm{diag}\{1, 5\}$, and the solution of the PDE (14) is derived as

$$\Psi(q,t) = [q_1 - \frac{q_3 \cos q_3 \cos t}{\tilde{m}^{1/2}}, q_2 \cos q_3 - q_1 \sin q_3, q_3]^{\top} \quad (26)$$

The data for the mobile robot are $d = 1.2$ m$, l = 1.2$ m$, r = 0.25$ m$, m = 1$ kg$, I = 0.2$ kg.m$^2, J = 1$ kg.m^2 Simulation results shown in Fig. 2 demonstrate smooth asymptotic convergence to the origin. Note that the common disadvantage of a time-varying time-periodic feedback is the lack of control over the speed of convergence [17].

B. Extension to coordinated stabilization

Consider a network of three mobile robots, where one is the leader designated as **L** and the others are followers designated as **1** and **2**. The leader robot is to be stabilized to the origin with the controller proposed in the last section. The task of formation control (with only relative pose feedback)

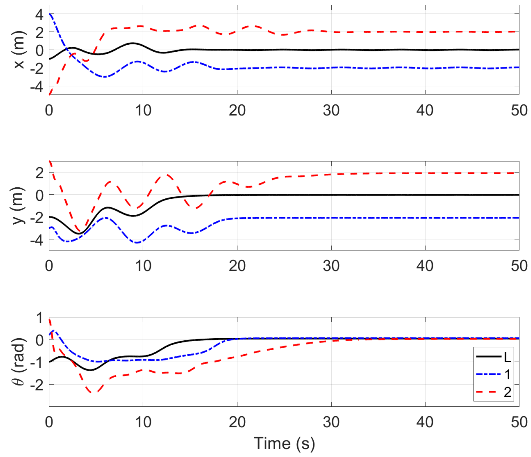

Fig. 3. Coordinated stabilization of a network of mobile robots.

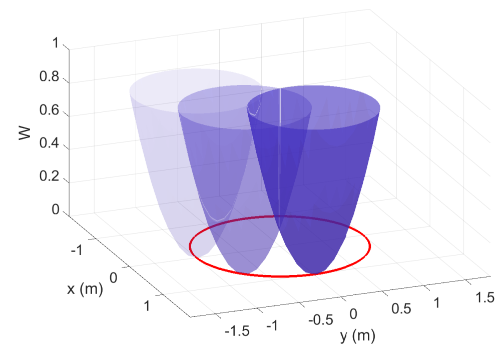

Fig. 4. Time-varying potential energy function moving counterclockwise along the desired trajectory.

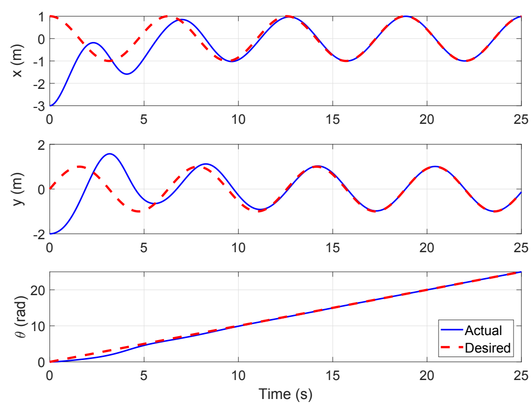

Fig. 5. Mobile robot actual versus desired pose trajectories.

is to make robots **1** and **2** follow the leader with relative poses $r_{\mathbf{1}}$ and $r_{\mathbf{2}}$, respectively, that is

$$\lim_{t\to\infty}(q_{\mathbf{i}}(t)-q_{\mathbf{L}}(t)-r_{\mathbf{i}})=0,\ \ \mathbf{i}=\mathbf{1},\mathbf{2}, \tag{27}$$

where $q_{\mathbf{L}}, q_{\mathbf{i}}$ are pose vectors of the leader and follower robot **i**, respectively. In the formation controller design for the follower robots, the potential energy shaping is based on their relative poses to the leader robot:

$$W_{\mathbf{i}}(\bar{q}_{\mathbf{i}})=\frac{1}{2}(\bar{q}_{\mathbf{i}}-\overline{q_{\mathbf{L}}-r_{\mathbf{i}}})^{\top}(\bar{q}_{\mathbf{i}}-\overline{q_{\mathbf{L}}-r_{\mathbf{i}}}), \tag{28}$$

where the bar represents the transformation of the variable according to equation (4). The coordinated stabilization results for relative poses $r_{\mathbf{1}}=[-2,-2,0]^{\top}$ and $r_{\mathbf{2}}=[2,2,0]^{\top}$ are shown in Fig. 3.

C. Trajectory tracking of a single robot

For trajectory tracking control design, we need to initially define the feasible trajectories. The reference trajectory $q_{\mathrm{d}}(t)=[x_{\mathrm{d}}(t),y_{\mathrm{d}}(t),\theta_{\mathrm{d}}(t)]^{\top}$ is forced to obey the same nonholonomic constraints as the mobile robot (24), that is, given any smooth functions $x_{\mathrm{d}}(t)$ and $y_{\mathrm{d}}(t)$, the desired orientation $\theta_{\mathrm{d}}(t)$ needs to satisfy the following constraint:

$$d\dot{\theta}_{\mathrm{d}}(t)=-\sin\theta_{\mathrm{d}}(t)\dot{x}_{\mathrm{d}}(t)+\cos\theta_{\mathrm{d}}(t)\dot{y}_{\mathrm{d}}(t). \tag{29}$$

From the generalized canonical transformation (11) and (13), we know that the energy will be dissipated and $\bar{H}$ will tend to zero because of the strict passivity of the closed-loop system, and p will tend to $-M^{1/2}\gamma(q,t)$. Therefore, in the trajectory tracking control design, $\gamma(q,t)$ needs to be selected such that $p\to p_{\mathrm{d}}(t)$, i.e., $\gamma(q,t)=-M^{-1/2}p_{\mathrm{d}}(t)$, leading to the generalized canonical transformation for kinetic energy shaping.

For the part of potential energy shaping, a time-varying function shown in Fig. 4 is selected. The quadratic time-varying potential energy function $W(\bar{q},\bar{q}_{\mathrm{d}}(t))=\frac{1}{2}(\bar{q}-\bar{q}_{\mathrm{d}}(t))^{\top}(\bar{q}-\bar{q}_{\mathrm{d}}(t))$ has a minimum that corresponds to the transformed desired trajectory $\bar{q}_{\mathrm{d}}(t)=\Psi(q_{\mathrm{d}}(t),t)$ at every instant t. Therefore, as the potential energy function tends to zero, the state q tends to the desired trajectory $q_{\mathrm{d}}(t)$. It should be noted here that the passivity condition (21) is related to desired trajectory since $\gamma(q,t)$ is determined by $p_{\mathrm{d}}(t)$, and in the case that $p_{\mathrm{d}}(t)$ is a constant vector (e.g. uniform motion), the passivity condition (21) can be guaranteed by setting $S=0$ similar to the stabilization case.

Next, suppose that the desired trajectory is $x_{\mathrm{d}}(t)=\cos(t), y_{\mathrm{d}}(t)=\sin(t)$, and the desired orientation $\theta_{\mathrm{d}}(t)$ can be obtained from (29). For the energy shaping of mobile robots, we choose $\gamma(q,t)=-M^{1/2}\dot{q}_{\mathrm{d}}(t)$ and $W(\bar{q},\bar{q}_{\mathrm{d}}(t))=0.5(\bar{q}-\bar{q}_{\mathrm{d}}(t))^{\top}(\bar{q}-\bar{q}_{\mathrm{d}}(t))$. As shown in Fig. 5, the states converge to the desired trajectory asymptotically. Fig. 6 shows the path of the mobile robot converging to the desired reference path.

D. Extension to coordinated trajectory tracking

Consider again the same network of a leader and two followers as in the stabilization problem. The formation control objective is to make the two followers follow the leader robot to form a specific formation, i.e.,

$$\lim_{t\to\infty}(q_{\mathbf{i}}(t)-q_{\mathbf{L}}(t)-r_{\mathbf{i}})=0,\ \ \mathbf{i}=\mathbf{1},\mathbf{2}. \tag{30}$$

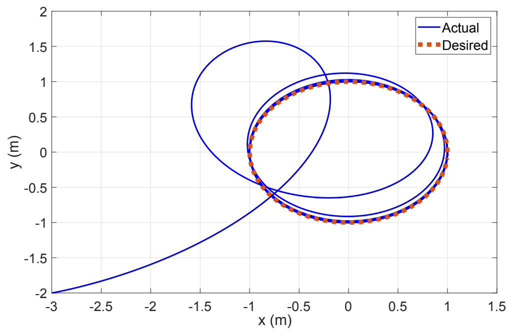

Fig. 6. Mobile robot actual versus desired path.

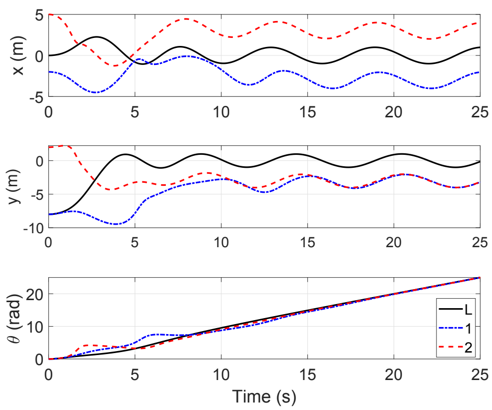

Fig. 7. Coordinated trajectory tracking of a network of mobile robots.

The leader control law is the same as the previous section. The potential energy shaping for the formation controller design of follower robots are based on their relative poses to the leader robot as shown in equation (28). Suppose that the relative poses in the formation are $r_1 = [-3, -3, 0]^\top$ and $r_2 = [3, -3, 0]^\top$. Note that the relative angles must be 0 to keep the formation satisfy the constraint (29). The simulation results of the coordinated trajectory tracking control are shown in Fig. 7 and the corresponding paths of the robots are shown in Fig. 8. Note that in trajectory tracking control, unlike the stabilization problem, the robots can be made to quickly converge to the desired formation.

V. CONCLUSIONS

In this paper, we developed an approach to design passivity-based controllers for planar underactuated vehicles with zero potential energy, such as nonholonomic mobile robots and surface vessels. The controller design techniques proposed in this paper can be used to achieve stabilization and trajectory tracking of underactuated planar vehicles with a unified structure. The method is further applied to coordinated control of a network of vehicles. Numerical simulations were presented to illustrate the effectiveness of the approach. Our future research is to applied this method to cooperative control of heterogeneous vehicles.

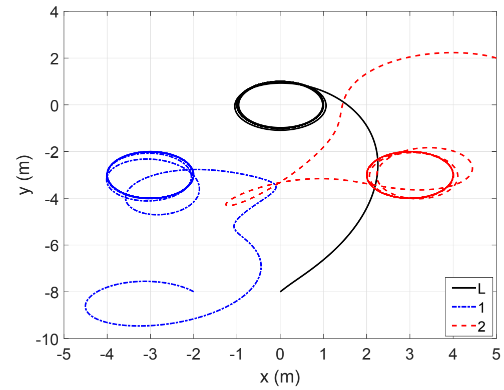

Fig. 8. Paths of the network of three mobile robots.

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