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Scaling-Based Reciprocal Control Barrier Functions for Nonholonomic Mobile Robots

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Why is obstacle avoidance challenging?

Force-controlled robot

$$\dot{p} = ve(\theta), \quad \dot{\theta} = \omega,$$

$$\dot{v} = u_1, \quad \dot{\omega} = u_2.$$

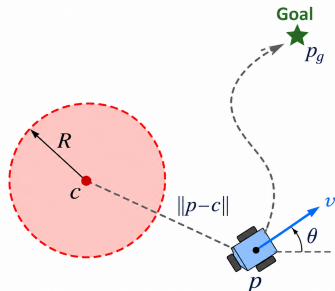
Physical safe set

$$h_0 = \|p - c\|^2 - R^2, \quad \mathcal{C}_0 = \{h_0 \geq 0\}.$$

First-order input dependence

$$L_g h_0 = 0.$$

Safety is specified in position; inputs act through acceleration.



Challenge: obtain an input-dependent CBF *without modifying the physical safety specification*.

Why not simply apply a standard CBF?

For $\dot{x} = f(x) + g(x)u$, let $\mathcal{C} = \{x : h(x) \geq 0\}$.

Standard (zeroing) CBF. A C^1 function h is a CBF on $S \supset \mathcal{C}$ if some $\alpha \in \mathcal{K}_\infty^e$ satisfies

$$\sup_{u \in \mathbb{R}^m} \{L_f h(x) + L_g h(x)u\} \geq -\alpha(h(x)), \quad x \in S.$$

For h_0 with $L_g h_0 = 0$, representative higher-relative-degree constructions include:

HOCBF (recursive auxiliary function)

$$\psi_1 = \dot{h}_0 + \alpha_0(h_0) \geq 0.$$

ReCBF (rectification)

$$H_R = h_0 - \mu \max\{0, -\psi_1\}.$$

Backstepping CBF (virtual-control error)

$$H_B = h_0 - \frac{1}{2\mu}(x_2 - \kappa(x_1))^2.$$

Interpretation: Such auxiliary conditions can restrict the certified domain for particular constructions.

Physical safety versus certified domain

E.g., Double integrator

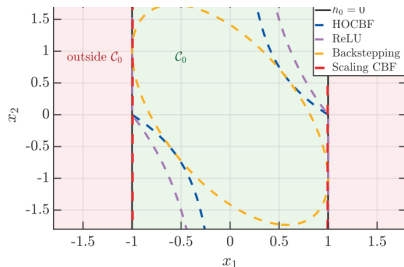
$$\dot{x}_1 = x_2, \quad \dot{x}_2 = u, \quad h_0 = 1 - x_1^2.$$

Representative additive construction

$$h = h_0 - \eta, \quad \eta \geq 0.$$

$$h \geq 0 \implies h_0 \geq \eta \geq 0.$$

The certified domain may be strictly smaller than the physical safe set $\mathcal{C}_0$.



Paper Fig. 1: representative certified domains.

Design goal: preserve $\text{Int}(\mathcal{C}_0)$ while restoring first-order control authority.

Why ordinary positive scaling is insufficient

Preserve the same safety geometry

$$h(x) = \sigma(x)h_0(x), \quad \sigma(x) > 0.$$

But the input coefficient becomes

$$L_g h = h_0 L_g \sigma.$$

On $h_0 = 0$, the control coefficient vanishes.

Positive multiplicative scaling retains the safe set, but loses first-order input authority at the boundary.

Reciprocal CBF: A barrier that diverges at the safety boundary and satisfies

$$\inf_{u \in \mathbb{R}^m} \{L_f B(x) + L_g B(x)u\} \leq \alpha_B \left(\frac{1}{B(x)} \right).$$

[e.g., $B(x) = \frac{1}{h(x)}$]

Our key idea: scaling in the reciprocal barrier

$$B(x) = \frac{\lambda(x)}{h_0(x)}, \quad 0 < \underline{\lambda} \leq \lambda(x) \leq \bar{\lambda} < \infty.$$

Denominator: physical boundary

$$h_0 \rightarrow 0^+ \implies B \rightarrow +\infty.$$

A positive, bounded numerator preserves the exact interior $\{h_0 > 0\}$.

Numerator: input dependence

Since $L_g h_0 = 0$,

$$L_g B = \frac{L_g \lambda}{h_0}.$$

Choose motion-dependent λ so that $L_g \lambda \neq 0$ throughout the interior.

Two separate roles: h_0 retains physical safety geometry; λ restores first-order control authority without imposing an auxiliary safe-set constraint.

Main result: scaling-based reciprocal CBF and safety

Motion-dependent numerator for the robot

$$\dot{e}_r = \frac{d}{dt}(\|p - c\| - R)$$

$$\lambda = \pi + \varepsilon_v \arctan(k_v \dot{e}_r) + \varepsilon_\omega \arctan(k_\omega \omega).$$

Nonvanishing control coefficient

$$\frac{\partial \lambda}{\partial \omega} = \frac{\varepsilon_\omega k_\omega}{1 + k_\omega^2 \omega^2} > 0.$$

Therefore $L_g B \neq 0$ on $\text{Int}(\mathcal{C}_0)$.

Reciprocal-CBF safety condition

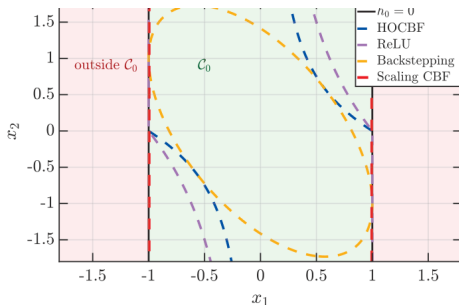
$$L_f B + L_g B u \leq \alpha_B(1/B).$$

Finite-time boundary contact would require $B \rightarrow \infty$, which the inequality prevents.

Theorem 1 + Corollary 1

B is a reciprocal CBF on the entire physical safe-set interior. Controllers satisfying the barrier condition prevent boundary contact; forward-complete closed-loop solutions preserve the safety for all time.

Example I: certified-domain comparison



Proposed double-integrator CBF

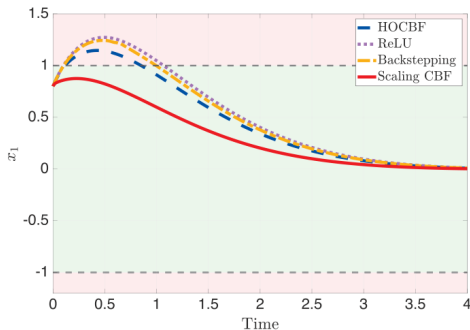
$$B = \frac{\lambda_0 + \varepsilon \arctan(k_v x_2)}{1 - x_1^2}.$$

Its domain covers every state with $|x_1| < 1$, irrespective of velocity.

The comparison demonstrates **certificate-domain preservation**, not universal performance superiority.

Paper Fig. 1; representative parameter choices.

Example I: closed-loop behavior



Initial condition

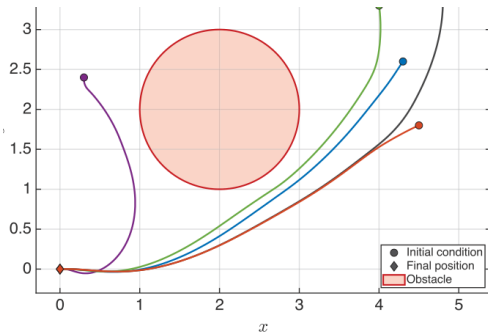
$$(x_1(0), x_2(0)) = (0.8, 2.5).$$

The initial state is physically safe but outside the comparison methods' certified domains.

The proposed trajectory stays safe; the comparison illustrates certificate coverage rather than universal controller ranking.

Paper Fig. 2; no input saturation.

Example II: obstacle avoidance



Circular obstacle

$$c = (2, 2)^T, \quad R = 1.$$

Five CLF-CBF-QP trajectories reach toward the origin while avoiding collision.

Minimum clearance:
0.2064.

The proposed constraint integrates with an optimization-based safety filter for the complete robot dynamics.

Paper Fig. 3; unconstrained inputs.

Conclusions

For force-controlled nonholonomic systems, we developed RCBF

$$B(x) = \frac{\lambda(x)}{h_0(x)}.$$

- ▶ Preserve the original physical safe-set interior.
- ▶ Restore nonzero first-order input dependence through motion-dependent scaling.
- ▶ Certify interior safety for forward-complete closed-loop solutions satisfying the barrier constraint.

Theorem 2 extends this construction to scalar strict-feedback cascades under explicit relative-degree conditions.

Thank you

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Theorem 2 for strict-feedback systems

Relative-degree- n cascade

$$\dot{x}_1 = f_1(x_1) + g_1(x_1)x_2,$$

$$\dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)x_3,$$

$$\vdots$$

$$\dot{x}_{n-1} = f_{n-1}(x_1, \dots, x_{n-1}) + g_{n-1}(x_1, \dots, x_{n-1})x_n,$$

$$\dot{x}_n = f_n(x) + g_n(x)u.$$

For $h_0 = \bar{h}(x_1)$, define $\Lambda = L_f^{n-1}h_0$.

$$B = \frac{\phi(\Lambda)}{h_0}, \quad L_g B = \frac{\phi'(\Lambda)L_g L_f^{n-1}h_0}{h_0} \neq 0.$$

Theorem 2

Assume relative degree n , nonvanishing input channels, and positive bounded ϕ with $\phi' \neq 0$. Then B is a reciprocal CBF for unconstrained inputs.

Guarantees and limitations

Established

- ▶ Full physical safe-set interior as the certificate domain.
- ▶ Nonzero first-order input coefficient.
- ▶ Safety under the stated feedback and completeness assumptions.

Limitations

- ▶ Bounded-input feasibility is not guaranteed.
- ▶ Near-boundary corrections can be large.
- ▶ No general superiority over other CBF constructions is claimed.

Trade-off: a larger certified state domain can require stronger control action.