



# A survey of underactuated mechanical systems

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**Abstract:** An underactuated mechanical system (UMS) is a system which has fewer independent control actuators than degrees of freedom to be controlled. Control of UMS is considered as one of the most active fields of research because of its diverse engineering applications. This survey reviews UMS from its history to the state-of-the-art research on modelling, classification, control, and to some extent, provides some unique insights for bottleneck issues and future research directions.

## 1 Introduction

An underactuated mechanical system (UMS) is a system which has fewer independent control actuators than degrees of freedom (DOF) to be controlled. This class of systems have been the subject of active scientific research fuelled by their broad applications in different disciplines. This survey aims to give a comprehensive but refined review of UMS from its history to the state-of-the-art research from the point of view of control, and focuses on its applications in robotics. To some extent, this paper provides some unique insights for classification of UMS, bottleneck issues of control and future research directions.

Studies on UMS can trace back to two decades ago when control of non-holonomic mechanical systems were of great interest by scientists [1, 2], and they generated interesting control problems [3, 4] which traditional control theory was not applicable. Examples of such systems include, but not limited to, mobile robot, spacecraft, underwater vehicle, surface vessel, helicopter, space robot and underactuated manipulator. UMS has attracted more attention in 1990s, and they have been widely used in aerospace and robotics, for example, [5, 6]. Control of UMS is now one of the most active research fields in control community.

Robots, mechatronics, and hybrid machines are extensively employed in engineering to replace labour force for economic manufacture. Aircrafts, satellites and space platforms are designed for exploration and military. Micro-robots, precise instruments and operational tools are developed to replace original medical tools for non-invasive diagnosis. Novel mechatronic systems are designed for necessary needs from industry and healthcare. As a result, the efforts of scientists and engineers led to the active research on non-linear control theory, optimal control, adaptive control and others. People are appreciating the substantial accomplishments that have been achieved [7, 8], and they are enjoying the benefits brought by machines. With this satisfaction,

stimulated by their appetencies of exploring, researchers are eager to solve new issues by creating new systems and theories. Coming back to UMS, the research of UMS becomes crucial, and the following questions have arisen naturally:

- Can we control UMS better using our current control techniques?
- Is there any novel UMS that can solve our problems in practice?

These two questions more or less motivate current research of UMS, and to some extent, they also imply that control of UMS is still a major open problem.

The remainder of this survey is organised as follows. In Section 2, modelling of UMS is reviewed, and some UMS examples are given. In Section 3, classifications of UMS are given from different points of view. In Section 4, control problems of UMS are discussed, and some popular control methods are reviewed. In Section 5, current challenges of UMS are discussed from both theoretical and practical points of view. Finally, concluding remarks and future research directions are drawn in Section 6.

## 2 Modelling

### 2.1 Equations of motion

The equations of motion of UMS used in most literatures were obtained from the simple Lagrangian mechanical system [9] expressed as below

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = F(q)u \quad (1)$$

where  $L = T - V$ ,  $T$  is the kinetic energy,  $V$  is the potential energy of the system,  $q \in \mathfrak{R}^n$  denotes the configuration vector,  $u \in \mathfrak{R}^m$  is the actuator input vector, and  $F(q) \in \mathfrak{R}^{n \times m}$  is

a non-square matrix of external forces. In [10, 11], (1) was rewritten as

$$\sum_j d_{kj}(q)\ddot{q}_j + \sum_{ij} \Gamma_{ij}^k(q)\dot{q}_i\dot{q}_j + g_k(q) = p_k^T F(q)u \quad (2)$$

where  $k = 1, 2, \dots, n$ ,  $p_k$  is the  $k$ th standard basis in  $\mathbb{R}^n$ ,  $d_{kj}$  is an element of inertia matrix,  $g_k(q) = \partial V(q)/\partial q_k$ , and  $\Gamma_{ij}^k(q)$  are called Christoffel symbols and is defined as

$$\Gamma_{ij}^k(q) = \frac{1}{2} \left( \frac{\partial d_{kj}(q)}{\partial q_i} + \frac{\partial d_{ki}(q)}{\partial q_j} - \frac{\partial d_{ij}(q)}{\partial q_k} \right) \quad (3)$$

The vector form of (1) can be written as

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = F(q)u \quad (4)$$

where  $D(q) \in \mathbb{R}^{n \times n}$  is the inertia matrix, which is a positive-definite symmetric matrix,  $c_{ij} = \sum_{k=1}^n \Gamma_{ij}^k(q)\dot{q}_k$  is an element of  $C(q, \dot{q})$ ,  $C(q, \dot{q})\dot{q} \in \mathbb{R}^n$  contains two types of terms involving  $\dot{q}_i\dot{q}_j$  that are called Centrifugal terms ( $i = j$ ) and Coriolis terms ( $i \neq j$ ), and  $G(q)$  is the gravity term. The simple Lagrangian mechanical system expressed by (4) can represent both fully-actuated mechanical system and UMS according to the column rank of  $F(q)$ . The simple Lagrangian mechanical system is called a fully-actuated mechanical system if  $m = \text{rank}(F) = n$ , or is called an UMS if  $m < n$ .

Assuming  $F(q) = [0 \ I_m]^T$ , the configuration vector can be partitioned without loss of generality as  $q = [q_u \ q_a]^T$ , where  $q_u \in \mathbb{R}^{n-m}$  and  $q_a \in \mathbb{R}^m$  denote a unactuated and actuated configuration vectors, respectively. After partitioning the inertia matrix  $D(q)$  in (4), the equations of motion of UMS can be reformed as

$$\begin{bmatrix} D_u \\ D_a \end{bmatrix} \begin{bmatrix} \ddot{q}_u \\ \ddot{q}_a \end{bmatrix} + \begin{bmatrix} C_u(q, \dot{q}) \\ C_a(q, \dot{q}) \end{bmatrix} \begin{bmatrix} \dot{q}_u \\ \dot{q}_a \end{bmatrix} + \begin{bmatrix} G_u(q) \\ G_a(q) \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix} \quad (5)$$

where  $D_u \in \mathbb{R}^{(n-m) \times n}$ ,  $D_a \in \mathbb{R}^{m \times n}$ ,  $C_u(q, \dot{q}) \in \mathbb{R}^{(n-m) \times n}$ ,  $C_a(q, \dot{q}) \in \mathbb{R}^{m \times n}$ ,  $G_u(q) \in \mathbb{R}^{n-m}$ , and  $G_a(q) \in \mathbb{R}^m$ . In [12, 13], the partitioned model was written as

$$\begin{bmatrix} D_{uu}(q) & D_{ua}(q) \\ D_{au}(q) & D_{aa}(q) \end{bmatrix} \begin{bmatrix} \ddot{q}_u \\ \ddot{q}_a \end{bmatrix} + \begin{bmatrix} h_u(q, \dot{q}) \\ h_a(q, \dot{q}) \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix} \quad (6)$$

where the vectors  $h_u(q, \dot{q}) \in \mathbb{R}^{n-m}$  and  $h_a(q, \dot{q}) \in \mathbb{R}^m$  contain centripetal, Coriolis and gravitational terms. In order to facilitate analysis and control of UMS, the following fundamental properties are obtained from (4):

- The inertial matrix  $D(q)$  is symmetric, uniformly positive definite, and bounded above and below, that is

$$0 < \alpha_m(q)I_n \leq D(q) \leq \alpha_M(q)I_n < \infty \quad (7)$$

where  $I_n$  is an  $n \times n$  identity matrix,  $\alpha_m(q)$  and  $\alpha_M(q)$  are scalar positive constants;

- The two matrices  $D(q)$  and  $C(q, \dot{q})$  are not independent. In particular, the Centrifugal and Coriolis terms  $C(q, \dot{q})\dot{q}$  is uniquely defined, but the matrix  $C(q, \dot{q})$  is not. So, using a proper definition of  $C(q, \dot{q})$ ,  $\dot{D}(q) - 2C(q, \dot{q})$  is skew-symmetric such that

$$X^T[\dot{D}(q) - 2C(q, \dot{q})]X = 0 \quad (8)$$

where  $X^T$  is the transpose of  $X \in \mathbb{R}^n$  which is an arbitrary vector [14];

- Defining the total energy of the system  $E(q, \dot{q})$ , for conservative systems, gives

$$E(q, \dot{q}) = T(q, \dot{q}) + V(q) = \frac{1}{2}\dot{q}^T D(q)\dot{q} + V(q) \quad (9)$$

then the following formula is obtained

$$\dot{E}(q, \dot{q}) = \dot{q}^T \left[ F(q)u - \frac{\partial P(\dot{q})}{\partial \dot{q}} \right] \quad (10)$$

where  $P(\dot{q})$  is the dissipation term of UMS.

## 2.2 Examples of UMS

This section presents several well-known examples of UMS, including the Acrobot, the Pendubot, the cart-pole system, the crane system, the rotating pendulum, the inertia-wheel pendulum (IWP), the beam-and-ball system, the magnetic suspension system, the translational oscillator with rotational actuator (TORA) system, the vertical take off and landing (VTOL) aircraft, and the surface vessel. All the examples are chosen because of the fact that for analysis and control purposes, the complexity of their control design are of great interests by researchers. The equations of motion of these UMS examples are unified in a vector form by (4), and their unique matrices are summarised in Table 1.

**2.2.1 Acrobot and Pendubot:** Acrobot [15] and Pendubot [16] are two-link manipulators with a single actuator at elbow and shoulder as shown in Fig. 1, respectively. Both manipulators graphically seem to be similar, and have the same equations of motion. The only difference is the input matrix  $F(q)$ , where  $F = [0 \ 1]^T$  is for the Acrobot, and  $F = [1 \ 0]^T$  is for the Pendubot. The control task for both systems is to stabilise the two-link manipulator to its upright equilibrium point ( $q_1 = \frac{\pi}{2}$  and  $q_2 = 0$ ) from any initial condition. One popular control approach is to use energy-based control to swing up the system from its stable downward position to unstable upright position, and switches to a linear controller for stabilisation [15–19]. Similar approach has been adopted by Lai *et al.* [20] to give a comprehensive unified control strategy for both two-link manipulators. Recent works by Albahkali *et al.* [21] and Jafari *et al.* [22] gave a fresh idea on swing-up control of these two systems through an impulse-momentum approach, but without exception of using energy. An interesting work done by Yabuno *et al.* [23] investigated the reachable and stabilisable areas of the Pendubot, and proposed a control strategy using high-frequency excitation under the circumstance when the state feedback of the second link was not available.

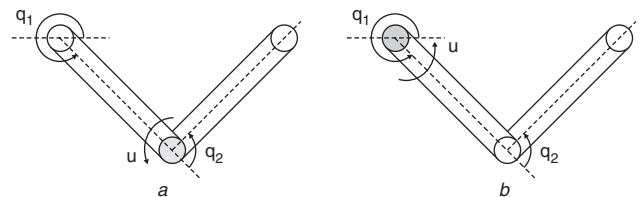


Fig. 1 Two-link manipulators

- a Acrobot
- b Pendubot

Table 1 Equations of motion of UMS examples

System	$D(q)$	$C(q, \dot{q})$	$G(q)$	$F(q)$
Acrobot	$\begin{bmatrix} m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c2}^2 + 2l_1 l_{c2} C_2) + I_1 + I_2 & m_2 (l_{c2}^2 + l_1 l_{c2} C_2) + I_2 \\ m_2 (l_{c2}^2 + l_1 l_{c2} C_2) + I_2 & m_2 l_{c2}^2 + I_2 \end{bmatrix}$	$\begin{bmatrix} -m_2 l_1 l_{c2} S_2 \dot{q}_2 & -m_2 l_1 l_{c2} S_2 (\dot{q}_1 + \dot{q}_2) \\ m_2 l_1 l_{c2} S_2 \dot{q}_1 & 0 \end{bmatrix}$	$\begin{bmatrix} (m_1 l_{c1} + m_2 l_1) g C_1 + m_2 l_{c2} g C_{12} \\ m_2 l_{c2} g C_{12} \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Pendubot	$\begin{bmatrix} m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c2}^2 + 2l_1 l_{c2} C_2) + I_1 + I_2 & m_2 (l_{c2}^2 + l_1 l_{c2} C_2) + I_2 \\ m_2 (l_{c2}^2 + l_1 l_{c2} C_2) + I_2 & m_2 l_{c2}^2 + I_2 \end{bmatrix}$	$\begin{bmatrix} -m_2 l_1 l_{c2} S_2 \dot{q}_2 & -m_2 l_1 l_{c2} S_2 (\dot{q}_1 + \dot{q}_2) \\ m_2 l_1 l_{c2} S_2 \dot{q}_1 & 0 \end{bmatrix}$	$\begin{bmatrix} (m_1 l_{c1} + m_2 l_1) g C_1 + m_2 l_{c2} g C_{12} \\ m_2 l_{c2} g C_{12} \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
cart-pole	$\begin{bmatrix} m_1 + m_2 & m_2 l C_1 \\ m_2 l C_1 & m_2 l^2 + I \end{bmatrix}$	$\begin{bmatrix} 0 & -m_2 l \dot{q}_1 S_1 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -m_2 g l S_1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
crane	$\begin{bmatrix} m_1 + m_2 & m_2 l C_1 \\ m_2 l C_1 & m_2 l^2 + I \end{bmatrix}$	$\begin{bmatrix} 0 & -m_2 l \dot{q}_1 S_1 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ m_2 g l S_1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
rotating pendulum	$\begin{bmatrix} I_1 + m_1 l_1^2 + m_2 (L_1^2 + l_2^2 S_2^2) & m_2 L_1 l_2 C_2 \\ m_2 L_1 l_2 C_2 & m_2 l_2^2 + I_2 \end{bmatrix}$	$\begin{bmatrix} \frac{1}{2} m_2 l_2^2 \dot{q}_2 S_{22} & -m_2 L_1 l_2 \dot{q}_2 S_2 + \frac{1}{2} m_2 l_2^2 \dot{q}_1 S_{22} \\ -\frac{1}{2} m_2 l_2^2 \dot{q}_1 S_{22} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -m_2 g l_2 S_2 \\ 0 \\ -(m_1 l_1 + m_2 L_1) g S_1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$
IWP	$\begin{bmatrix} I_1 + I_2 + m_1 l_1^2 + m_2 L_1^2 & I_2 \\ I_2 & I_2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
magnetic suspension	$\begin{bmatrix} m & 0 \\ 0 & \frac{m^2}{k_1 q_1 + k_2} \end{bmatrix}$	$\begin{bmatrix} 0 & \dot{q}_2 \\ \frac{k_1 m^2}{2(k_1 q_1 + k_2)^2} & -\dot{q}_2 - \dot{q}_1 \end{bmatrix}$	$\begin{bmatrix} -mg \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
beam-and-ball	$\begin{bmatrix} I + I_b + m d_2^2 & 0 \\ 0 & I_b / r^2 + m \end{bmatrix}$	$\begin{bmatrix} m q_2 \dot{q}_2 & m q_2 \dot{q}_1 \\ -m q_2 \dot{q}_1 & 0 \end{bmatrix}$	$\begin{bmatrix} m g q_2 C_1 \\ m g S_1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
TORA	$\begin{bmatrix} m_1 + m_2 & m_2 r C_2 \\ m_2 r C_2 & I + m_2 r^2 \end{bmatrix}$	$\begin{bmatrix} 0 & -m_2 r \dot{q}_2 S_2 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} k q_1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
VTOL	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ g \\ 0 \end{bmatrix}$	$\begin{bmatrix} -S_3 & \epsilon C_3 \\ C_3 & \epsilon S_3 \\ 0 & 1 \end{bmatrix}$
surface vessel	$\begin{bmatrix} m C_3 & m S_3 & 0 \\ -m S_3 & m C_3 & 0 \\ 0 & 0 & I \end{bmatrix}$	$\begin{bmatrix} \xi_1 C_3 & \xi_1 S_3 & 0 \\ -\xi_2 S_3 & \xi_2 C_3 & 0 \\ 0 & 0 & \xi_3 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ d & -d \\ \frac{d}{2} & -\frac{d}{2} \end{bmatrix}$

$S_j := \sin(q_j)$ ,  $C_j := \cos(q_j)$ ,  $S_{ij} := \sin(q_i + q_j)$ ,  $C_{ij} := \cos(q_i + q_j)$

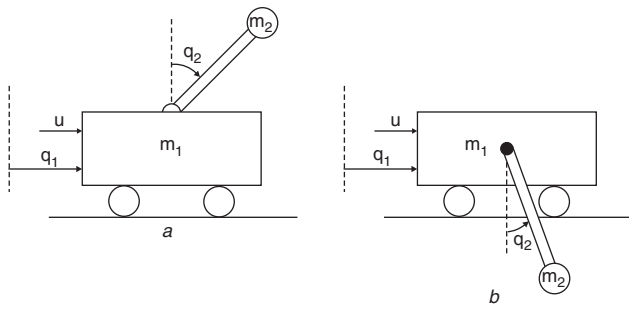


Fig. 2 Cart-pole and crane systems

- a Cart-pole system
- b Crane system

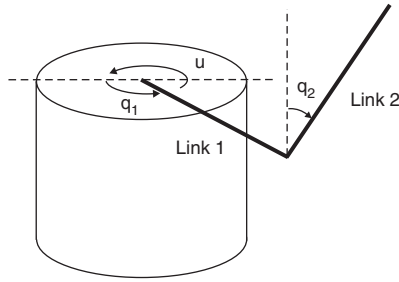


Fig. 3 Rotating pendulum

**2.2.2 Cart-pole and crane systems:** The cart-pole system shown in Fig. 2a is a benchmark underactuated system that has been widely used as a test bed for non-linear control research. The control task is to swing up the pendulum from its stable downward equilibrium position ( $q_1 = 0$  and  $q_2 = \pi$ ) to upright unstable equilibrium position ( $q_2 = 0$ ), while keeps the cart at its original position ( $q_1 = 0$ ). The control works of the cart-pole system in the past two decades are huge from energy-based method to fuzzy control (FC) approach [17, 24–28], and its extended systems are massive, for example, wheeled inverted pendulum [29–31], two-parallel-pendulum cart [32], two-serial-pendulum cart [33], three-dimensional (3D) pendulum cart [34], pendulum-driven cart [35, 36] and double-pendulum-driven cart [37]. Similar to the equations of motion of the cart-pole system, the crane system shown in Fig. 2b has a totally different control objective. The control task of the crane system is to reduce undesired payload swing by controlling the motion of the trolley. Some recent works on motion control of the crane system can be referred to [38–41].

**2.2.3 Rotating pendulum:** The rotating pendulum (is also called Furuta pendulum) shown in Fig. 3 is an inverted pendulum (Link 2) connecting to a rotating arm (Link 1). The control objective of the system is to swing up Link 2 from its stable downward position ( $q_2 = \pi$ ) to upright unstable position ( $q_2 = 0$ ) by rotating Link 1, and balance it upright thereafter. To control the rotating pendulum, there is no exception of using energy-based control methods, for example, [42–45]. An interesting work done by Freidovich *et al.* [46, 47] considered the problem of creating oscillations of the rotating pendulum rather than stabilising it at upright position.

**2.2.4 IWP:** The IWP shown in Fig. 4 consists of an unactuated pendulum with an actuated rotating uniform inertia wheel at its end. The IWP is a flat underactuated system with two-DOF and a single actuator acting on the wheel,

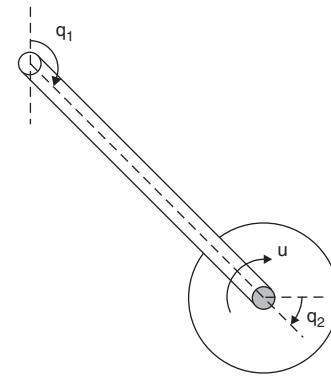


Fig. 4 IWP

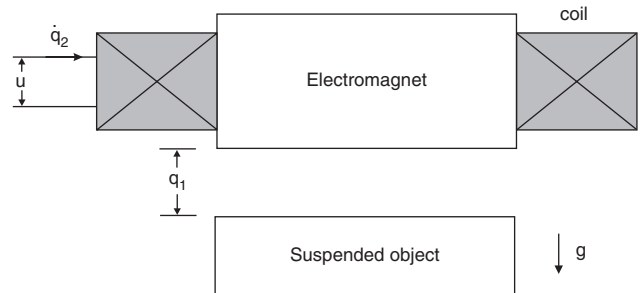


Fig. 5 Magnetic suspension system

and the control task is to swing up the pendulum by rotating the wheel, and stabilises it at its upright equilibrium position ( $q_1 = 0$ ) thereafter. The control methods for IWP include interconnection and damping assignment-passivity-based control (IDA-PBC) [48], saturation technique [49], backstepping [50] and sliding mode control (SMC) [51].

**2.2.5 Magnetic suspension system:** The magnetic suspension system shown in Fig. 5 is a typical non-linear underactuated system that consists of an electromagnet, a suspended object, and the coil. The magnetic suspension system has been widely used in industrial applications, for examples, magnetic bearings, levitated vehicle, etc. Since its behaviour is highly non-linear and unstable, a feedback control system with high performance is always desirable. The control task is to move the suspended object from its initial position to a desired position under external disturbance. The control work on magnetic suspension system can be referred to [52–54].

**2.2.6 Ball-and-beam system:** The ball-and-beam system [55] shown in Fig. 6 contains a beam rotating on a vertical plane, an actuator mounted on the joint connecting the beam to the base, and a ball rolling freely along the beam. The control task is to move the ball from any initial position with any initial speed to a desired position on the beam by applying the torque to the beam. The well-established control works on the ball-and-beam system include IDA-PBC [48],  $\lambda$ -method [56], a comparison of linear quadratic regulator, subspace stabilisation and combined error metric approaches [57], energy-based control [45] and Lyapunov-based method [58].

**2.2.7 TORA system:** The TORA system shown in Fig. 7 is a non-linear benchmark example for the purpose of comparison and test of different control strategies. The system contains a translational oscillating platform and a

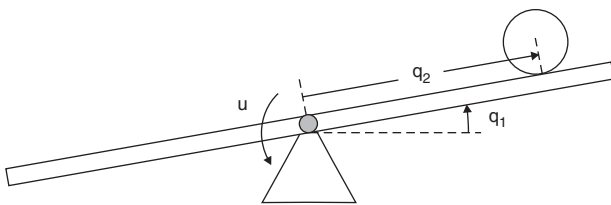


Fig. 6 Ball-and-beam system

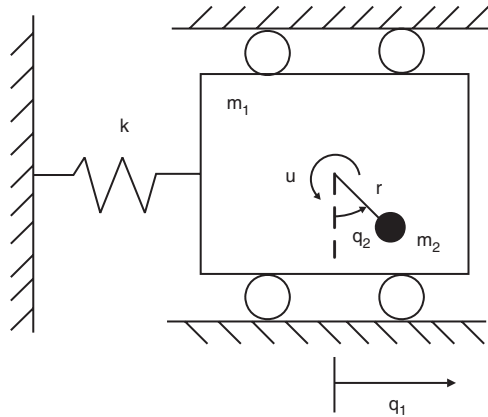


Fig. 7 TORA system

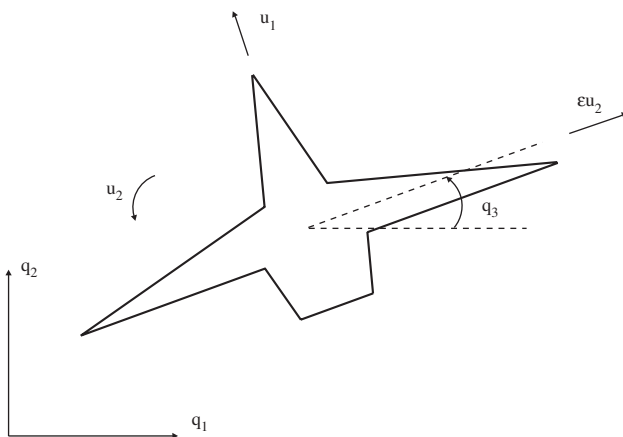


Fig. 8 VTOL aircraft

rotational eccentric pendulum. The control task is to find a control law such that the horizontal displacement  $q_1$  tends to zero in the presence of any external disturbance, and some control works can be referred to [59–64].

**2.2.8 VTOL aircraft:** The simplified VTOL aircraft depicted in Fig. 8 is a UMS with three-DOF and two control inputs. Under the assumption that  $|\varepsilon|$  is relatively small, it is a slightly non-minimum phase system [65]. When  $|\varepsilon| \gg 1$ , it is a strongly non-minimum phase system that stabilisation of such a system becomes difficult [66]. Most of the existing research on VTOL aircraft has focused on set-point regulation [67–71] and trajectory tracking [72–79].

**2.2.9 Surface vessel:** The surface vessel shown in Fig. 9 is a simplified model of underactuated surface ships with 3-DOF and two independent thrust inputs which have received considerable attention from the control community recently. The bulk of existing research has focused on global

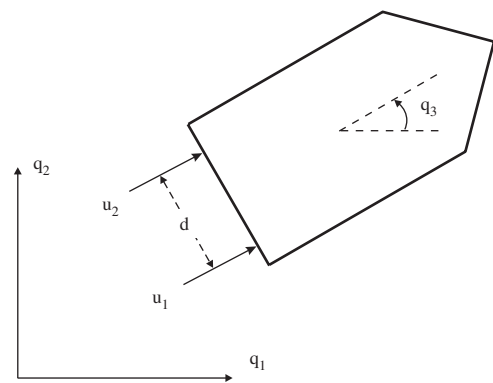


Fig. 9 Surface vessel

stabilisation and tracking control [80–87], and with a special concern to its parametric uncertainties [88–90].

### 3 Classification of UMS

This section classifies UMS based on different system properties, and according to the redundancy and the fact that some of these properties have been less studied for practical applications, this paper only focuses on some particular ones that frequently appeared in the literatures.

#### 3.1 Classification by the reasons of underactuation

It is known that the challenging issue of UMS is to control the system by using fewer independent actuators than DOF. In order to address this issue, it is necessary to understand the reasons that cause underactuation. The following reasons were summarised in [10] which can be used to classify UMSs:

- Dynamics of the system by nature, for example, spacecraft, helicopter, underwater vehicle [91] and wheeled mobile robot [92, 93];
- By design for reduction of the cost or some practical purposes, for example, satellites with two thrusters [94, 95], flexible-link robot [96];
- Actuator failure, for example, surface vessel, VTOL aircraft;
- Imposed artificially to create complex low-order nonlinear systems for the purpose of gaining insight into controlling high-order UMS, for example, the cart-pole system, the beam-and-ball system, the Acrobot and the Pendubot.

#### 3.2 Classification by system constraint

Many UMSs are subject to their first-order or second-order constraints. The first-order constraint is the constraint including generalised coordinate and velocity, and the second-order constraint is the constraint including generalised coordinate, velocity and acceleration. The differential constraint is holonomic if it is integrable, and is non-holonomic if it is non-integrable. An underactuated system is holonomic if its constraint is completely integrable, and is non-holonomic if its constraint is non-integrable or partial integrable [97]. Examples of holonomic and non-holonomic UMSs are given in Table 2.

**Table 2** Classification by system constraint

Class	Constraint	Examples
holonomic UMS	completely integrable	horizontal pendulum [98], two-link planar manipulator [97], pendulum-driven cart
non-holonomic UMS	partial integrable or non-integrable	wheeled mobile robot, surface vessel, Acrobot, Pendubot

**Table 3** Classification by configuration characteristics [10]

Class	(a)	(b)	(c)	(d)	Examples
I	yes <sup>a</sup>	yes	yes	no	Acrobot, IWP, TORA
IIa	no	yes	yes	no	rotating pendulum, Pendubot, beam-and-ball
IIb	no	yes	yes	yes	flexible-link robot
III	no	yes	yes	yes	cart-pole system
IVa	yes	no	yes	no	controlled VTOL aircraft <sup>b</sup>
IVb	yes	no	yes	yes	controlled VTOL aircraft
V	yes	no	yes	yes	VTOL aircraft, helicopter
VIa	yes	yes	no	no	three-link pendulum <sup>c</sup>
VIb	yes	yes	no	yes	three-link pendulum
VIIa	no	yes	no	no	three-link pendulum
VIIb	no	yes	no	yes	three-link pendulum
VIII	no	yes	no	yes	3D cart-pole system

<sup>a</sup>A 'Yes' or a 'No' means the corresponding property holds or does not hold, respectively

<sup>b</sup>A controlled VTOL aircraft means the dynamics of the VTOL (in 3-DOF with two control inputs) in closed loop with a feedback  $u_1 = u_1(q_1, q_2, \dot{q}_1, \dot{q}_2)$  and a single remaining control  $u_2$

<sup>c</sup>The three-link pendulum is a 3-DOF system with two control inputs, and the actuation configuration may lead the system to different classes

**Table 4** Classification by control problems

Class	Control problems	Examples
car-like	trajectory planning and tracking	wheeled mobile robot, surface vessel, VTOL aircraft, helicopter, biped robot
pendulum-like	set-point regulation	cart-pole, Acrobot, Pendubot, IWP

### 3.3 Classification by configuration characteristics

UMSs can be classified by the configuration characteristics [10] as follows: (a) actuated shape variables, (b) non-interacting inputs, (c) integrable normalised momentums and (d) extra required conditions (details can be referred to [10]). The descriptions and examples of different classes of UMSs are given in Table 3.

### 3.4 Classification by control problems

The control problems of underactuated manipulators have been discussed in [99] which can be extended to control of UMS as below:

- Trajectory planning: given an initial configuration  $q_0$  and a final desired configuration  $q_d$ , find a dynamically feasible trajectory that joins  $q_0$  and  $q_d$ ;
- Trajectory tracking: given a dynamically feasible trajectory  $q_d(t)$ , find a feedback control law that stabilises the tracking error  $e(t) = q_d(t) - q(t)$  to zero asymptotically;
- Set-point regulation: given a desired configuration  $q_d$ , find a feedback control law that stabilises the equilibrium state  $q = q_d$  and  $\dot{q} = 0$ .

Based on the control problems above, UMSs can be classified into two categories as given in Table 4.

## 4 Control of UMS

Control of UMS aims to find a feedback control law that stabilises the system in the presence of various uncertainties and external disturbances, and such a wide research field can be partitioned into two main categories: set-point regulation and trajectory tracking. The reason behind the complexity of this field is that the number of actuators is less than the DOF to be controlled, and many traditional non-linear control methods are not directly applicable. In recent years, many control methods based on feedback linearisation, energy, backstepping, sliding mode, and fuzzy logic have been developed for stabilisation of UMS. This section provides a view to highlight some of these popular control methods.

### 4.1 Partial feedback linearisation

A useful technique for control of UMS is the so-called partial feedback linearisation (PFL) [24], which is a method

providing a natural global change of coordinates that transforms the system into a strict feedback form, and the control method can be easily applied to the new form of the system. There are two PFL techniques presented as below.

**4.1.1 Collocated PFL [17]:** Consider the actuated configuration vector  $q_a$  in (6), there exists a global invertible change of control in the form

$$u = \alpha_1(q)\tau + \beta_1(q, \dot{q}) \quad (11)$$

that partially linearises (6) as follows

$$\begin{aligned} \dot{q}_u &= p_u \\ \dot{p}_u &= f_0(q, p) + g_0(q)\tau \\ \dot{q}_a &= p_a \\ \dot{p}_a &= \tau \end{aligned}$$

over the set

$$U_1 = \{q \in \mathfrak{R}^n \mid \det(D_{uu}(q)) \neq 0\}$$

where  $q = [q_u \ q_a]^T$ ,  $\alpha_1(q)$  is an  $m \times m$  positive-definite symmetric matrix, and

$$\begin{aligned} f_0(q, p) &= -D_{uu}^{-1}(q)h_u(q, \dot{q}) \\ g_0(q) &= -D_{uu}^{-1}(q)D_{ua}(q) \end{aligned}$$

Collocated PFL shows a procedure of linearisation of actuated configuration variables which globally transforms all underactuated systems in the form of (6) to a fully-actuated form where the new control input appears in the dynamics of both subsystems. Olfati-Saber [10] further introduced a global change of coordinates that can decouple these two subsystems but left the linear subsystem invariant. The method has been extensively applied to control of the Acrobot [15], the three-link pendulum [17], the cart-pole system [28], the double pendulum cart [33], the pendulum-driven cart [36] and the capsule system [100].

**4.1.2 Non-collocated PFL [10]:** Consider the following UMS

$$\begin{bmatrix} d_{00}(q) & d_{01}(q) & d_{02}(q) \\ d_{10}(q) & d_{11}(q) & d_{12}(q) \\ d_{20}(q) & d_{21}(q) & d_{22}(q) \end{bmatrix} \begin{bmatrix} \ddot{q}_0 \\ \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} h_0(q, \dot{q}) \\ h_1(q, \dot{q}) \\ h_2(q, \dot{q}) \end{bmatrix} = \begin{bmatrix} u_0 \\ u_1 \\ 0 \end{bmatrix} \quad (12)$$

where  $q = [q_0 \ q_1 \ q_2]^T$ . Then there exists a change of control in the form

$$u = \alpha_1(q)\tau + \beta_1(q, \dot{q})$$

where  $u = [u_0 \ u_1 \ 0]^T$ ,  $\tau = [\eta \ \sigma \ 0]^T$  and

$$\sigma = \alpha_0(q)\eta + \alpha_2(q)\varphi + \beta_2(q, \dot{q})$$

that partially linearises (12) as below

$$\begin{aligned} \dot{q}_0 &= p_0 \\ \dot{p}_0 &= \eta \\ \dot{q}_1 &= p_1 \\ \dot{p}_1 &= f_0(q, p) + g_0(q)\eta + g_2(q)\varphi \\ \dot{q}_2 &= p_2 \\ \dot{p}_2 &= \varphi \end{aligned}$$

over the set

$$U = \{q \in \mathfrak{R}^n \mid \det(d_{21}(q)) \neq 0\}$$

where

$$\begin{aligned} f_0(q, p) &= -d_{21}^{-1}(q)h_2(q, p) \\ g_0(q) &= -d_{21}^{-1}(q)d_{20}(q) \\ g_2(q) &= -d_{21}^{-1}(q)d_{22}(q) \end{aligned}$$

and  $\varphi$  is the new control input.

Non-collocated PFL presents a procedure of linearisation of unactuated configuration variables, and also holds for a restricted class of underactuated systems under the case that the number of unactuated configuration variables is less than or equal to the number of control inputs. Olfati-Saber [101] studied non-collocated PFL for underactuated systems with symmetry property, and transformed the non-triangular quadratic normal form into a cascade non-linear system in a strict feedforward form. This method has been applied to control of the Pendubot [102], the rotating pendulum [101], the flexible one-link robot [103] and the surface vessel [104, 105]. The advantage of collocated and non-collocated PFL is a conceptual and structural simplification of control problem, that is, they are always utilised as an initial simplifying step for reduction and control of UMS.

**4.1.3 Remark on flat UMS:** A flat UMS is a system which has a constant inertia matrix,  $D(q)$  in (4), and  $C(q, \dot{q}) = 0$  because of the vanishing Christoffel symbols in (3). The flat UMS has limited controllability as its gravity term of unactuated part,  $G_u(q) \neq 0$  in (5). IWP and VTOL aircraft are systems in this class, and their specific global change of coordinates [10] is given as below.

Consider a flat UMS with configuration vector  $q = [q_x \ q_s]^T$  and a force matrix which is independent of  $q_x$ . Assume the potential energy of the system is independent of  $q_s$  (i.e.  $V(q) = V(q_x)$ ). The equations of motion for this system are written as

$$\begin{bmatrix} m_{xx} & 0 \\ 0 & m_{ss} \end{bmatrix} \begin{bmatrix} \ddot{q}_x \\ \ddot{q}_s \end{bmatrix} + \begin{bmatrix} -g_x(q_x) \\ 0 \end{bmatrix} = \begin{bmatrix} F_r(q_s) & F_x(q_s) \\ 0 & F_s(q_s) \end{bmatrix} \begin{bmatrix} \tau_r \\ \tau \end{bmatrix} \quad (13)$$

where  $g_x(q_x) = -\nabla_{q_x} V(q_x)$ ,  $\tau_r \in \mathfrak{R}$ ,  $F_r(q_s) : \mathfrak{R}^m \rightarrow \mathfrak{R}^{n-m}$  is a unit vector that is over a unit ball in  $\mathfrak{R}^{n-m}$ ,  $\tau \in \mathfrak{R}^m$ ,  $m_{xx}$  and  $m_{ss}$  are constant, and  $F_s(q_s)$  is an  $m \times m$  invertible matrix.

Let

$$\tau = F_s(q_s)^{-1}m_{ss}u$$

be the partially linearising change of control. Assume all the elements of

$$\omega = m_{xx}^{-1}F_x(q_s)F_s^{-1}(q_s)m_{ss} \cdot dq_s$$

are exact one-forms and let  $\omega = d\gamma(q_s)$ . Then the following global change of coordinates

$$\begin{aligned} q_r &= q_x - \gamma(q_s) \\ p_r &= m_{xx}p_x - F_x(q_s)F_s^{-1}(q_s)m_{ss}p_s \end{aligned}$$

with  $(p_x, p_s) = (\dot{q}_x, \dot{q}_s)$  transforms (13) into

$$\begin{aligned} \dot{q}_r &= m_r^{-1}p_r \\ \dot{p}_r &= g_r(q_r + \gamma(q_s)) - p_s^T \pi_F(q_s)p_s + F_r(q_s)\tau_r \end{aligned}$$

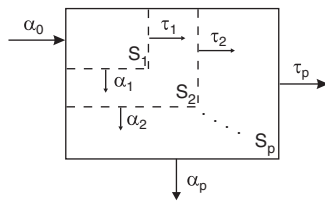


Fig. 10 Schematics of backstepping

$$\begin{aligned} \dot{q}_s &= p_s \\ \dot{p}_s &= u \end{aligned}$$

where  $\pi_F(q_s)$  is a cubic matrix satisfying

$$\frac{d}{dt}(F_x(q_s)F_s^{-1}(q_s))m_{ss}p_s = p_s^T \pi_F(q_s)p_s$$

## 4.2 Energy-based methods

4.2.1 Introduction to passivity: Considering the derivative of the total energy in (10) gives

$$\dot{E}(q, \dot{q}) = \dot{q}^T \left[ F(q)u - \frac{\partial P(\dot{q})}{\partial \dot{q}} \right] \leq \dot{q}^T \cdot F(q)u \quad (14)$$

which implies that the system is passive with respect to the input  $u$  and output  $\dot{q}$ . The passivity is a vital character of UMS since a passive UMS has a stable origin and a feedback control law always exists for  $\dot{E}(q, \dot{q}) \leq 0$ .

The control methods based on the passivity of UMS are referred to PBC. The main idea of these methods is to regulate the total energy of the system to the equivalent value of a desired equilibrium state by using (14). PBC is mostly used for set-point regulation of UMS, such as two-link manipulators [20], cart-pole [106], two-serial-pendulum cart [33], two-parallel-pendulum cart [32], rotating pendulum [47], TORA [107] and biped robot [108]. The main drawback of PBC is its narrow range of applications as very little practical applications in robotics and aerospace engineering have been developed so far.

There are many energy-based control methods, such as  $L_2$ -gain method [109, 110] and direct Lyapunov method [111, 112], but the following review will focus on backstepping, IDA-PBC and controlled Lagrangian methods according to their popularity.

4.2.2 Backstepping: One limitation of PBC is that it is only applicable to the systems with relative degree less than two. To overcome this limitation, a technique called backstepping is proposed to transform the system into a new recursive form which PBC can be easily applied to. The principle of backstepping [113] as depicted in Fig. 10 is to design a sequence of ‘virtual’ systems  $S_i$  ( $i = 1, 2, \dots, p$ ) of relative degree one, finishing with the actual system as the last member of the sequence. For each ‘virtual’ system  $S_i$ , the relative degree is reduced to one by selecting an available signal  $\alpha_{i-1}$  as a ‘virtual’ input and then achieving passivity with respect to a ‘virtual’ output  $\tau_i$ . Finally, the actual control law  $u = \alpha_p$  is obtained.

The backstepping technique is effective for global stabilisation of the UMS with low-DOF. In recent years, it has been extensively applied to control surface vessel [84, 87, 114–117] and VTOL aircraft [66, 76, 118, 119]. However, when the DOF of UMS increases, the procedure

of backstepping becomes very complicated, and implementation of such a control design in practical applications is unrealistic.

4.2.3 IDA-PBC and controlled Lagrangian methods: The Hamilton equation and the Euler-lagrange equation are two traditional ways to describe the behaviour of dynamic systems, and their systems are named as the Hamilton system and the Euler–Lagrange system, respectively. There are also two main PBC methods for both UMSs: IDA-PBC [48, 120, 121] and controlled Lagrangian [122, 123]. As it is known, the equilibrium state of a Hamilton/Euler–Lagrange system is related to a critical state of the Hamiltonian/Lagrangian. Both passivity-based methods consist of two stages: (i) shape the Hamiltonian/Lagrangian to a desired form with suitable equilibrium states and structural features by using control inputs, and then (ii) inject damping into the system to ensure the passivity of the system. A main difference between these two methods is that the desired Lagrangian in the controlled Lagrangian method is obtained by modifying only the generalised inertia matrix and the potential energy function, while IDA-PBC also considers the possibility of changing the interconnection matrix [48].

## 4.3 Sliding mode control

Uncertainty is a common yet intractable issue for the control of UMS. For any control design, there are typical discrepancies between a practical system and its theoretical model because of unmodelled dynamics, parameter uncertainty, and external disturbance. Adaptability and robustness to these factors has been an attractive research topic of control engineering in the past few decades. Robust control [124] and adaptive control [125] are two main techniques for uncertainty compensation, where the former is designed to ensure the system to be insensitive to all uncertainties by using a fixed control structure, whereas the later utilises on-line identification of which either system parameter or controller parameter. In the earlier times, robust and adaptive control of the UMSs with uncertainties were considered by numbers of researchers. For instance, Gu [126] proposed a normal-form augmentation approach for recovering the parameter-linearity of the system, and then applied a direct adaptive control scheme to stabilise the UMS with parameter uncertainty. Shin [127] proposed a robust adaptive control method for underactuated manipulators with free-flying passive joints. Su and Stepanenko [128] studied a model-based adaptive variable structure control scheme for set-point regulation of underactuated manipulators. Behal *et al.* [80] designed a continuous time-varying adaptive tracking controller for a surface vessel in the presence of uncertainties associated with hydrodynamic damping coefficients. Do *et al.* [129] developed a non-linear robust adaptive control strategy to control a surface vessel in the presence of environmental disturbances. However, owing to the nature of these two control structures, robust control is only suitable for dealing with small uncertainty [130], while adaptive control is suitable for a wide range of parameter variation, but is sensitive to unstructured uncertainty.

To solve these limitations, a versatile variable-structure controller called SMC has received considerable devotion from researchers in recent years. SMC is insensitive to parameter variation and external disturbance, since the behaviour on the sliding mode depends on the switching

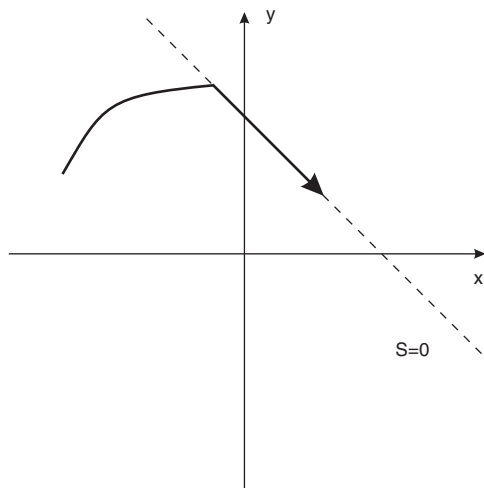


Fig. 11 Schematics of SMC

surface only, not the structural properties of the system. The basic idea of SMC is to alter the dynamics of the system by applying a discontinuous control input that forces the system to ‘slide’ along a predefined state surface, and the system produces a desired behaviour by restricting its state to this surface. A schematics of SMC is shown in Fig. 11, and its mathematical description is briefly given as follows.

Consider a second-order system in cascade normal form

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= f(x, y, t) + B(x, t)u(t)\end{aligned}$$

where  $(x, y)$  is the system state,  $f(x, y, t)$  and  $B(x, t)$  are generally non-linear functions, and  $u(t)$  is the control input. Define a positive-definite Lyapunov function

$$V(t) = \frac{1}{2}s^2 \quad (15)$$

where  $s(t)$  is a time-varying surface. An SMC exists in the form such that

$$u = u_{eq} + u_{sw} \quad (16)$$

satisfying

$$\dot{V} = s \cdot \dot{s} \leq 0 \quad (17)$$

where  $u_{eq}$  is the equivalent control for maintaining the system state along the sliding surface ( $s = 0$ ), and  $u_{sw}$  is the switching control for driving the system states towards to the sliding surface.

SMC has succeeded in the applications of control of underactuated satellite [131], surface vessel [85, 132], helicopter [133], wheeled inverted pendulum [31], ball-and-beam system [134], underactuated fuel cell system [135], TORA [136], underactuated biped robot [137] and overhead crane [138]. Meanwhile, many researchers also devoted themselves to design universal SMC for the UMSs that have the same dynamic characteristics, for example, [71, 98, 139–144]. Although SMC has grown exponentially in the past two decades, its applications on control of UMS is only mature at the level of simulation. Therefore it is worth noting that ‘the real test for the sliding mode research community in

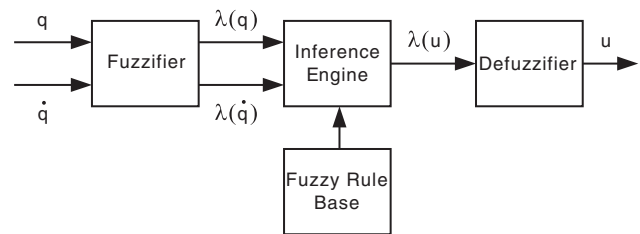


Fig. 12 Schematics of FC

the near future will be the willingness of control engineers to experiment with these SMC design approaches in their professional practice [145].’

#### 4.4 Fuzzy control

Generally, there exists a mismatch between human and machine control, that is, human factors result in uncertain, imprecise and fuzzy, while machine and computer control are accurate, efficient but unadaptable. As an artificial intelligent technique, FC is a way to make machines more intelligent enabling them to behave in a fuzzy manner like human. Proposed by Zadeh [146], FC emerged in 1965 as a tool to deal with uncertain, imprecise, or qualitative decision-making problems. As shown in Fig. 12, FC utilises linguistic information to model the qualitative aspects of human knowledge and reasoning processes without employing precise quantitative analysis. It is considered as a non-mathematical control approach for the system that cannot be well defined or modelled [147], and has been extensively used in the applications of UMS control. In general, FC has heuristics-based and model-based approaches, and the later (e.g. Takagi–Sugeno model-based approach) is more popular. There are plenty of research papers using model-based approaches, which readers can refer to, for example, for set-point regulation [27, 148–151], for tracking control [93, 152–155] and for FC combining with some other control techniques [156–159].

#### 4.5 Optimal control

Strictly speaking, optimal control is a control problem rather than a method, but the authors would refer it to different control approaches for optimisation in this paper. The objective of optimal control is to find a control law  $u(t)$  that minimises (or maximises) the cost function

$$J = \int f(q(t), \dot{q}(t), u(t)) \cdot dt \quad (18)$$

Optimal control problems mainly can be considered into two categories: energy-optimal and time-optimal. For energy-optimal control, the issue concerns the energy consumption of the system, for example, [160–163]. For time-optimal control, most of the control approaches were based on differential geometric methods for either a class of non-holonomic rigid systems [164–167] or some specific applications [168, 169], or even, many different approaches have been developed for the same problem, for example, spacecraft maneuvers [170–172]. Similar to energy-optimal control, there has not been a general approach for time-optimal control of UMS. Therefore, a general optimal control theory for UMS like the linear optimal control theory would be appreciated.

## 5 Current challenges of UMS

Highly non-linear characteristics and reduced dimension of input space are intractable issues for controlling UMS. The control methods were designed either to adapt to non-linearities or to reduce the system order to a low-dimensional model. However, not all of these methods are practically applicable because of system constraints (e.g. actuator power limitation). This section reviews these issues from the points of view of theoretical and practical challenges.

### 5.1 Theoretical challenges

**Controllability and stabilisation:** As a class of strongly non-linear systems, interest in UMS always concentrates on its complex dynamics, non-holonomic constraint, and lack of feedback linearisation, which classical linear control theory is not applicable. Although the dynamics of UMS has been well understood, more concerns have been made on its controllability. Some early works [173, 174] have shown that the controllability of a system is dependent on its linearisation property, that is, a system is locally controllable if its linearisation at an equilibrium is controllable. Then these studies were mainly limited to non-holonomic systems satisfying non-integrable kinematics relations [1, 175] with an extension to the UMSs that had non-integrable dynamics [176]. The important work done by Reyhanoglu *et al.* [176] showed that strong accessibility of UMS was a necessary condition for small-time local controllability [173] at an equilibrium, but it was insufficient to prove the existence of an asymptotically feedback stabilisation. Luca *et al.* [12] discussed sufficient conditions of small-time local controllability for UMS, which suggested to use either a discontinuous or a time-varying feedback controller for stabilisation as time-invariant continuous feedback control was not applicable for UMS, for example, [97, 177]. This more or less explains the reason that discontinuous control methods (e.g. SMC) are of great interests by researchers at present.

Recent works on the stabilisation of UMS using discontinuous feedback control have been developed for some specific UMSs or a specific class of UMSs rather than the entire framework. For example, Xu and Özgüner [71] proposed a global stabilisation scheme for a class of UMSs in cascaded form. Sankaranarayanan and Mahindrakar [143] developed a control approach which can globally stabilise a class of underactuated mobile systems that were not linearly controllable and violated Brockett's necessary condition [178]. Moreover, semi-global and global stabilisations were also done in [44, 98, 139, 179] for both set-point regulation and trajectory tracking control problems. A special concern is paid to the set-point regulation of UMS as there is no breakthrough on global stabilisation, and almost all the works (e.g. [3, 16, 28, 33]) were done by supervisory switching from swing-up control to balancing control which was actually not a global stabiliser. Global stability results maybe only exist for a limited class of UMSs. Thus, it is recommended to focus on specific classes of UMSs or a single UMS in order to make progress, for example, [20, 44, 87, 180].

**Configuration characteristics:** A notable work indebted to Olfati-Saber [10] who proposed an explicit change of coordinates that transformed several classes of UMSs into cascade non-linear systems with structural properties. The work was carried out under the configurations of UMS with actuated/unactuated shape variables, input coupling,

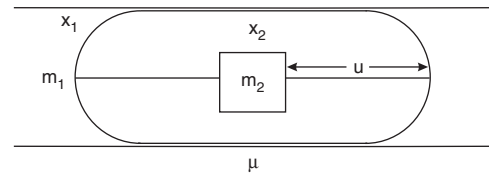


Fig. 13 Capsubot in a tube (adopted from [183])

and non-integrable momentums which had led to a detailed classification in Table 3. The merit of this work is that UMSs can be explicitly transformed into a strict feedback form, feedforward form, and non-triangular linear/quadratic form based on their configuration characteristics for which relevant control methods can be applied to easily. Some example applications of the work can be found in [66, 87, 101, 181, 182].

The constraint of UMS is another configuration characteristic that should draw reader's attention. As shown in Table 2, UMSs can be classified into holonomic and non-holonomic based on the integrability of their constraints. Almost all the research of UMS so far is on control of non-holonomic UMSs. However, dramatically rising interests have been paid to a class of holonomic underactuated robots [35, 37, 100, 183–186] recently because of their broad applications, and they have brought many interesting control issues. An example of this type is illustrated here to discuss some new control issues. An underactuated robot, the so-called Capsubot, is shown in Fig. 13, where an inner mass  $m_2$  interacts with a capsule main body  $m_1$  through an internal force  $u$  in the presence of dry friction with static friction coefficient  $\mu$ . The dynamic model of the system can be written as

$$\begin{bmatrix} m_1 & m_2 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} \mu(m_1 + m_2)g \cdot \text{sign}(\dot{x}_1) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix} \quad (19)$$

where  $g$  is the acceleration because of gravity. The Capsubot is a flat UMS with 2-DOF and a single control input, and it is also holonomic as its second-order constraint is completely integrable when  $\dot{x}_1 \neq 0$ . The control task of the system is to drive the entire system to a desired equilibrium, which can be considered as a control problem of set-point regulation. Then the issue emerges because of its non-smooth friction term which raises the difficulty level of control. On the other hand, this term is vital, because the system will not progress without it. Up to now, only open-loop control [183] and semi-closed-loop control [100] (i.e. closed-loop control of subsystem) are available, and the closed-loop control of UMS with non-smooth dynamics is still an open issue. Some recent works can be referred to [36, 37, 98, 187].

### 5.2 Practical challenges

**Industrial needs:** Nowadays, UMSs are widely used in aerospace and industry for exploration and manufacture. Examples of these systems include ship, underwater vehicle, helicopter, aircraft, satellite, mobile robot, space platform and flexible joint robot, etc. Although these examples have received many well-established successes in well-structured environment, their control techniques for complex uncertain environment are still immature [7, 8]. However, there is always a desiderative expectation of new applications of UMSs that requires autonomous operation in an unstructured and possibly dynamically changing environment [188].

Thus, control techniques that satisfy these needs become crucial.

**High-DOF complex UMS:** The benefits brought by UMS can be reduced weight, cost, and energy consumption, as well as maintaining adequate degrees of dexterity without losing reachable configuration space. They also have no or less damage when hitting an object, and more tolerance for actuator failure. As the DOF of UMS increases, the reliability of the system declines which results in practical problem of control design. Examples of such high-DOF complex systems include the systems involving an extremely large number of control loops, the coordination of a large number of autonomous agents to control a non-linear hybrid stochastic system, or very large model uncertainties [7]. These systems cannot work under hazardous or unsafe environment without any considerable reliability, since the costs of damage and loss are extremely high. Robustness to noise, disturbance, and uncertainty secures the reliability of UMS, and a need of new tools for addressing robustness issue is highly desirable.

**Fault-tolerant detection and control:** Actuator failure may cause either a reduction of the number of control inputs which converts a fully-actuated system to a UMS [189], or converts a UMS with more control inputs to a UMS with less control inputs. Sensor failure results in delayed or missing feedback signal which affects overall tracking performance of the system. Therefore both failures are to be considered by scientists and engineers in the design of UMS, and a fault detection mechanism is highly recommended, for example, [190–193].

**Networked UMS:** Another practical concern is networked control of UMS. Once network is introduced, communication package delay or loss may lead to poor control performance, since the communication between actuator and sensor can be interrupted. Control of UMS should be robust to network delay, and a network control technique (e.g. predictive control) is required.

## 6 Concluding remarks and future research

UMS is such an important class of mechanical systems that have been applied in diverse engineering applications. Control of UMS is one of the most active fields of research in control community. For such a wide research area, it is difficult to include all the works of UMS in a review paper. However, we believe that this survey could provide reader with the most up-to-date knowledge in the field of UMS. Finally, several important issues and possible future research directions are drawn:

- **Set-point regulation:** As a mature and well-defined control issue, set-point regulation for a class of UMSSs, such as Acrobot, Pendubot and cart-pole system, has been extensively studied. However, there are still potential contributions for this matter, for example, recent works [21, 22, 168, 194, 195], which proposed new set-point regulation methods;
- **Tracking control:** Compared to fruitful achievements for fully-actuated systems, tracking control of UMS is still a challenge topic. Some recent works [71, 143, 179] can be referred;
- **Modelling with friction:** Although friction plays a significant role in some precise applications [11, 100], it is always ignored [4, 10], or simplified [93, 184] in modelling of UMS.

Some papers [98, 196–198] can give readers an open mind along this direction;

- **Novel UMS:** Some UMS examples, such as biped robot [199, 200], robotic hand [201], pendulum-driven cart [36, 37], Capsubot [100] and AcroBOX [202], should bring reader's attention as design of new UMS is one of the trends in this field. We believe it is a promising research direction.

## 7 Acknowledgment

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