ME 41100 System Dynamics and Control

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Homework 7

Problem 1. Give the state description matrices in control-canonical form for the following transfer functions:

(1) $G(s) = \dfrac{1}{4s+1}$

(2) $G(s) = \dfrac{2s+1}{s^2+3s+2}$

(3) $G(s) = \dfrac{s+3}{s(s^2+2s+2)}$

Problem 2. A certain system with state x is described by the state matrices,

$$A = \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 3 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \end{bmatrix},\ D = 0$$

Find the transformation T so that if $z = Tx$, the state matrices describing the dynamics of z are in control canonical form. Compute the new matrices $\bar{A}, \bar{B}, \bar{C}$, and $\bar{D}$.

Problem 3. Consider the plant described by

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 3 \end{bmatrix} x$$

(1) Find the transfer function using matrix algebra.

(2) Design a state feedback controller that satisfies the following specifications: damping ratio $\xi = 0.707$, and step-response peak time is under 3.14 sec.

(3) Verify your design with Matlab using the functions `ss(A,B,C,D)` and `step`.

Problem 4. Consider the transfer function

$$\frac{Y(s)}{U(s)} = \frac{s}{s^2 + 4}$$

(1) Write a set of equations that describes this system in the standard controller canonical form as $\dot{x} = Ax + Bu$, $y = Cx$.

(2) Design a control law of the form $u = -k_1 x_1 - k_2 x_2$ which will place the closed-loop poles at $s = -2 \pm 2j$.

Problem 5. Consider a system given by

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 2 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

(1) Determine if it is possible to assign arbitrary pole locations for the system.

(2) It is desired to have eigenvalues at 3 and 5 by using a state-feedback control $u = Kx$. Determine the necessary feedback gain matrix K and the control law u.