Safety-Critical Stabilization of Force-Controlled Nonholonomic Mobile Robots

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Safety-critical stabilization

Problem: **Stabilize** the robot at the target, while guaranteeing that the trajectory remains within a **safe space**

In Cartesian Coordinates

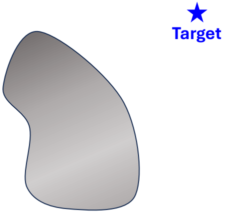
$$\text{kinematics:} \begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\theta} = \omega \end{cases}$$

$$\text{kinetics:} \begin{cases} \dot{v} = u_v \\ \dot{\omega} = u_\omega \end{cases}$$

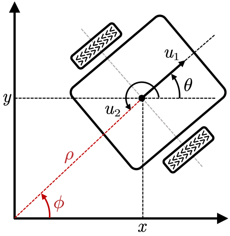
In Polar Coordinates

$$\text{kinematics:} \begin{cases} \dot{\rho} = v\cos\alpha \\ \dot{\phi} = \dfrac{v}{\rho}\sin\alpha \\ \dot{\alpha} = \omega - \dfrac{v}{\rho}\sin\alpha \end{cases}$$

$$\text{kinetics:} \begin{cases} \dot{v} = u_v \\ \dot{\omega} = u_\omega \end{cases}$$



Obstacle

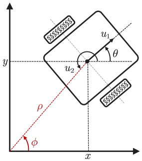


Construction of strict Lyapunov function

Method: Control Lyapunov function + control barrier function + QP

[Nominal controller] in polar coordinates:

$$
\begin{aligned}
v^* &= -k_\rho \cos(\alpha)\rho \\
\omega^* &= -k_\alpha \alpha - k_\rho \operatorname{sinc}(2\alpha)(\alpha - \lambda\phi) \\
u_v &= \dot{v}^* - \rho(k_\rho \cos(\alpha)^2 z - \cos(\alpha) z^2 + k_z z) \\
u_\omega &= \dot{\omega}^* - k_\omega \tilde{\omega}
\end{aligned}
$$



[Closed-loop system] in cascaded structure:

$$
\begin{bmatrix} \dot{\rho} \\ \dot{\phi} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} -k_\rho \cos(\alpha)^2 \rho \\ -k_\rho \operatorname{sinc}(2\alpha)\alpha \\ -k_\alpha \alpha + \lambda k_\rho \operatorname{sinc}(2\alpha)\phi \end{bmatrix} + \begin{bmatrix} \rho\cos\alpha & 0 \\ \sin\alpha & 0 \\ -\sin\alpha & 1 \end{bmatrix} \begin{bmatrix} z \\ \tilde{\omega} \end{bmatrix}
$$

$$
\dot{z} = -k_z z, \quad \dot{\tilde{\omega}} = -k_\omega \tilde{\omega}.
$$

[Strict Lyapunov function] is given by

$$
\mathcal{V}_r(\rho, \phi, \alpha, z, \tilde{\omega}) := \mu \ln(V(\rho, \phi, \alpha) + 1) + U(z, \tilde{\omega})
$$

$$
V(\rho, \phi, \alpha) := \frac{\nu(r)}{2}(\rho^2 + \lambda\phi^2 + \alpha^2) + \xi^\top P \xi, \quad U(z, \tilde{\omega}) := \frac{1}{2}\left(\frac{z^2}{k_z} + \frac{\tilde{\omega}^2}{k_\omega}\right)
$$

Safety-critical control design

Method: Control Lyapunov function + control barrier function + QP

[Integrator backstepping] to construct CBF in Cartesian coordinates:

$$\begin{aligned}\Sigma_1 : \dot{x}_1 &= f(x_1) + g(x_1)x_2 \\ \Sigma_2 : \dot{x}_2 &= u\end{aligned}$$

Suppose we know a CBF B_1 for Σ_1 and a "virtual controller" x_2^*.
With $\tilde{x}_2 := x_2 - x_2^*(x_1)$, the system becomes

$$\begin{aligned}\Sigma_1 : \dot{x}_1 &= f_{\mathsf{safe}}(x_1) + g(x_1)\tilde{x}_2 \\ \Sigma_2 : \dot{\tilde{x}}_2 &= u - \dot{x}_2^* =: \tilde{u}.\end{aligned}$$

Then the CBF is given by

$$B(\mathbf{x}) := B_1(x_1) + \tilde{x}_2^\top H \tilde{x}_2, \quad H = H^\top > 0$$

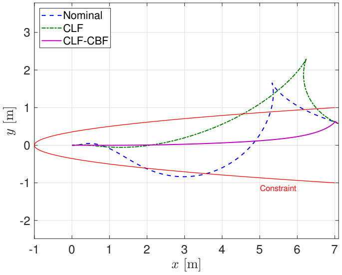
[Quadratic programming] (γm-version) integrates CLF and CBF

$$\begin{aligned}\min \quad & \frac{1}{2}(\bar{u}^\top \bar{u} + m\delta^\top \delta) \\ \text{s.t.} \quad & F_1 := \gamma_f(L_{f_1}\mathcal{V}_r + \alpha(|\chi|)) + L_{g_1}\mathcal{V}_r\bar{u} + L_{g_1}\mathcal{V}_r\delta \leq 0 \\ & F_2 := L_{f_2}B(\mathbf{x}) - \alpha_B\left(1/B(\mathbf{x})\right) + L_{g_2}B(\mathbf{x})\bar{u} \leq 0\end{aligned}$$

Illustrative examples

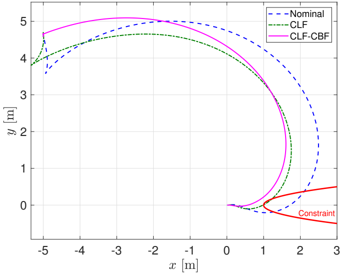
Main result: local asymptotic stability + safety

Example 1



Safety region enforcement

Example 2



Obstacle avoidance

To prove further:

- T. Han and B. Wang, "Safety-Critical Stabilization of Force-Controlled Nonholonomic Mobile Robots," in *IEEE Control Systems Letters*, vol. 8, pp. 2469-2474, 2024.
- G. Wang, B. Wang, Q. Xu and J. Wang, "Further Results on Safety-Critical Stabilization of Force-Controlled Nonholonomic Mobile Robots," in *arXiv*, July 2025.

Thank you!

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https://bwang-ccny.github.io/